Testing Statistical Hypotheses: Two Sample Problem

The difference in the means of Two Populations – Two independent Samples

Normal Populations σ_1^2 and σ_2^2 known

Null Hypothesis: H_0	Alternative Hypothesis: H_1	Test Statistic	Rejection Region	p-value
$\mu_1 - \mu_2 = d_0$	$\mu_1 - \mu_2 \neq d_0$	$z = \dfrac{\overline{x}_1 - \overline{x}_2 - d_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	$\lvert z\rvert > z_{\alpha/2}$	$2P(Z > \lvert z\rvert)$
$\mu_1 - \mu_2 \geq d_0$	$\mu_1 - \mu_2 < d_0$		$z < -z_\alpha$	$P(Z < z)$
$\mu_1 - \mu_2 \leq d_0$	$\mu_1 - \mu_2 > d_0$		$z > z_\alpha$	$P(Z > z)$

σ_1^2 and σ_2^2 unknown, large samples

Null Hypothesis: H_0	Alternative Hypothesis: H_1	Test Statistic	Rejection Region	p-value
$\mu_1 - \mu_2 = d_0$	$\mu_1 - \mu_2 \neq d_0$	$z = \dfrac{\overline{x}_1 - \overline{x}_2 - d_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$	$\lvert z\rvert > z_{\alpha/2}$	$2P(Z > \lvert z\rvert)$
$\mu_1 - \mu_2 \geq d_0$	$\mu_1 - \mu_2 < d_0$		$z < -z_\alpha$	$P(Z < z)$
$\mu_1 - \mu_2 \leq d_0$	$\mu_1 - \mu_2 > d_0$		$z > z_\alpha$	$P(Z > z)$

Normal Populations σ_1^2 and σ_2^2 unknown, $\sigma_1^2 = \sigma_2^2$ small samples

Null Hypothesis: H_0	Alternative Hypothesis: H_1	Test Statistic	Rejection Region	p-value
$\mu_1 - \mu_2 = d_0$	$\mu_1 - \mu_2 \neq d_0$	$t = \dfrac{\overline{x}_1 - \overline{x}_2 - d_0}{s_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$	$\lvert t\rvert > t_{\alpha/2,f}$	$2P(t_f > \lvert t\rvert)$
$\mu_1 - \mu_2 \geq d_0$	$\mu_1 - \mu_2 < d_0$		$t < -t_{\alpha,f}$	$P((t_f < t)$
$\mu_1 - \mu_2 \leq d_0$	$\mu_1 - \mu_2 > d_0$		$t > t_{\alpha,f}$	$P((t_f > t)$

where $$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

and f, the number of degrees of freedom, $= n_1 + n_2 - 2$.

The difference in the means of Two Populations – Two related Samples (Matched Pairs)

Null Hypothesis: H_0	Alternative Hypothesis: H_1	Test Statistic	Rejection Region	p-value
$\mu_1 - \mu_2 = d_0$	$\mu_1 - \mu_2 \neq d_0$	$t = \dfrac{\overline{D} - d_0}{s_D / \sqrt{n}}$	$\lvert t\rvert > t_{\alpha/2,n-1}$	$2P(t_{n-1} > \lvert t\rvert)$
$\mu_1 - \mu_2 \geq d_0$	$\mu_1 - \mu_2 < d_0$		$t < -t_{\alpha,n-1}$	$P((t_{n-1} < t)$
$\mu_1 - \mu_2 \leq d_0$	$\mu_1 - \mu_2 > d_0$		$t > t_{\alpha,n-1}$	$P((t_{n-1} > t)$

The difference between two proportions

Conditions:

n_1 and n_2 large,

$n_1\pi_1 \geq 5 \qquad n_1(1-\pi_1) \geq 5$

$n_2\pi_2 \geq 5 \qquad n_2(1-\pi_2) \geq 5$

Null Hypothesis: H_0	Alternative Hypothesis: H_1	Test Statistic	Rejection Region	p-value
$\pi_1 - \pi_2 = d_0$	$\pi_1 - \pi_2 \neq d_0$	$z = \dfrac{p_1 - p_2 - d_0}{\sqrt{\bar{p}(1-\bar{p})\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$, where $p_1 = \dfrac{X_1}{n_1}$, $p_2 = \dfrac{X_2}{n_2}$ and $\bar{p} = \dfrac{X_1 + X_2}{n_1 + n_2}$	$\lvert z\rvert > z_{\alpha/2}$	$2P(Z > \lvert z\rvert)$
$\pi_1 - \pi_2 < d_0$	$\pi_1 - \pi_2 < d_0$		$z < -z_{\alpha}$	$P(Z < z)$
$\pi_1 - \pi_2 > d_0$	$\pi_1 - \pi_2 > d_0$		$z > z_{\alpha}$	$P(Z > z)$

Tests for Variances: Normal Populations

Null Hypothesis: H_0	Alternative Hypothesis: H_1	Test Statistic	Rejection Region
$\sigma_1^2 = \sigma_2^2$	$\sigma_1^2 \neq \sigma_2^2$	$F = \dfrac{S_1^2}{S_2^2}$, with $n_1 - 1$ and $n_2 - 1$ d.f. and S_1^2 is the larger sample variance	$F > F_{\alpha/2}$
$\sigma_1^2 \leq \sigma_2^2$	$\sigma_1^2 > \sigma_2^2$		$F > F_{\alpha}$