February 06,2013

QUIZ♯1 Math102, sec 7

Net Time Allowed: 25 minutes

Name: **ID ♯:** **Serial:**

Exercise1: (04pts)

Let f be a continuous function on $[-\frac{3}{2}, x]$ with $f(-\frac{3}{2}) = 0$, F, G two differentiable functions such that:

$$F(x) = \int_{-\frac{3}{2}}^{x} f(t)\,dt \quad \text{and} \quad G(u) = \int_{-\frac{3}{2}}^{u} e^{-t} F(t)\,dt.$$

Find $G'(-\frac{3}{2}) + G''(-\frac{3}{2})$.

Solution:

- $G'(u) \overset{0.5}{=} e^{-u} F(u) \Rightarrow G'(-\frac{3}{2}) = e^{\frac{3}{2}} F(-\frac{3}{2}) = e^{\frac{3}{2}} \times 0 = 0$ (0.5)
- $G''(u) \overset{0.5}{=} -e^{-u} F(u) + e^{-u} F'(u)$ or $F'(u) \overset{0.5}{=} f(u)$, Thus

$G''(u) \overset{0.5}{=} -e^{-u} F(u) + e^{-u} f(u) \Rightarrow G''(-\frac{3}{2}) \overset{0.5}{=} 0 + e^{\frac{3}{2}} f(-\frac{3}{2}) = 0$ ($f(-\frac{3}{2}) = 0$)

Hence $\boxed{G'(-\frac{3}{2}) + G''(-\frac{3}{2}) = 0}$ (01)

Exercise2: (06pts)

Evaluate the integrals:

1- $\int \frac{x-1}{\sqrt{1-x^2}}\,dx$ (03pts)

2- $\int \sin^5(\frac{x}{3}) \cos(\frac{x}{3})\,dx$ (03pts)

1- $\int \frac{x-1}{\sqrt{1-x^2}}\,dx \overset{01}{=} \int \frac{x}{\sqrt{1-x^2}}\,dx - \int \frac{dx}{\sqrt{1-x^2}} = -\sqrt{1-x^2} - \sin^{-1}x + C$ (01) (01)

2- $\int \sin^5(\frac{x}{3}) \cos(\frac{x}{3})\,dx \overset{02}{=} \int 3u^5\,du = \frac{3}{6}u^6 + C = \frac{1}{2}\sin^6(\frac{x}{3}) + C$ (01)

$u = \sin(\frac{x}{3})$, $du = \frac{1}{3}\cos(\frac{x}{3})\,dx$