King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Mid Term Exam: Math 571

Name:.....ID:.....

**Exercise 1:** Consider the linear equation

$$y' = P_1(t)y + Q_1(t)$$

a) Show that the derivatives needed in the Taylor series algorithm can be obtained from the recurrence relation,

$$y^{(r)} = P_r(t)y + Q_r(t)$$

were

$$\begin{cases} P_r(t) = P'_{r-1}(t) + P_1(t)P_{r-1}(t) \\ Q_r(t) = Q'_{r-1}(t) + Q_1(t)P_{r-1}(t) \end{cases}$$

for $r = 2, 3, ...$

b) Apply the result of (a) to obtain a third order Taylor series formula to approximate the solution to the initial value problem

$$\begin{cases} y' = ty + 1 \\ y(0) = 0 \end{cases}$$

at $t = 1$ with step size $h = 0.25$ and find the error knowing that the 'exact' value of the solution at $t = 1$ is $y(1) = 1.4106861346$.

**Exercise 2:** Derive Runge-Kutta methods of order two (Mid-point, modified Euler, Henn's) and four for the IVP

$$\begin{cases} y' = -y + t \\ y(0) = 1 \end{cases}$$

**Exercise 3:** Let $y' = f(y)$,
a) Find $y'', y'''$
b) Set up a correspondence $F$ between trees and elementary differentials
Find $F(\quad)(y), F(\quad)(y), F(\quad)(y), F(\quad)(y)$
c) Find $\frac{d}{dt}F(\quad)(y)$