King Fahd University of Petroleum & Minerals
Department of Math. & Stat.

Math 568 - Midterm Exam (122)
Time: 3 hours 00 mns

Wednesday, April 11, 2013

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Name: ID #
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Problem 1	/10
Problem 2	/10
Problem 3	/10
Problem 4	/15
Problem 5	/5
Total	/45

Problem # 1. (10 marks) Given

$$\begin{aligned} &x^2u_x + y^2u_y = u^2 \\ &u(x, 2x) = x^2, \quad x \in \mathbb{R} \end{aligned} \qquad (i)$$

a. Find all the characteristic points in

$$\Sigma = \{(x, 2x),\ x \in \mathbb{R}\}$$

b. Use the characteristic method to solve the problem (i)

Problem # 2. (10 marks) Use the characteristic method to solve

$$u_x^2 + u_y = 0, \qquad u(x,0) = x$$

Problem # 3. (10 marks) In $\Omega = (0,1) \times (-1,1)$, show that $u(x,y) = x|y|$ is a weak solution for the equation

$$u_x + 2u_y = (y + 2x)\text{sign}y$$

Problem # 4. (15 marks) For $x > 0, y > 0$, let

$$xu_{xx} - yu_{yy} + \frac{1}{2}(u_x - u_y) + \sqrt{x}u_x + \sqrt{y}u_y = 0, \qquad (*)$$

a. Show that the PDE (*) is hyperbolic
b. Show that, by a convenient change of variable, that (*) reduces to

$$w_\xi - w_{\xi\eta} = 0, \qquad (**)$$

b. Solve (**) and then find the general solution $u(x, y)$.

Problem # 5. (5 marks) Discuss the uniqueness of the following problem

$$\begin{aligned} \Delta u(x) - \int_{\Omega} u(y)dy &= f(x), \qquad \text{in } \Omega \\ u &= \varphi, \qquad \text{on } \partial\Omega \end{aligned}$$