King Fahd Univ. of Petroleum and Minerals
Faculty of Sciences
Department of Mathematics and Statistics

FINAL EXAM
MATH. 531-122

Name:

ID:

Prob. 1
(a) Show that if $0 < W < \infty$, then $e^{1/x} \notin \mathbb{L}^p(0, w)$ for any p $(0 < p \leq \infty)$.
(b) Show that $(1 + |\ln x|)^{-1} x^{-1/2} \in \mathbb{L}^p(0, \infty)$ if $p = 2$ but not otherwise.

Prob. 2
(a) Show that $f_n \to f$ in measure and $g_n \to g$ in measure imply that $f_n + g_n \to f + g$ in measure.
(b) Show that if $\mu(E) < \infty$ and $f_n \to f$ in measure then $f_n^2 \to f^{\,2}$ in measure.

Prob. 3
Discuss the convergence (pointwise, a.e., uniform, a-uniform, uiform a.e., in the mean of order p) for
(a) $f_n(x) = n^{1/p} e^{-nx}$, $p > 0$, $0 \leq x \leq 1$,
(b) $f_n(x) = n^{3/2} x e^{-n^2 x^2}$, $0 \leq x \leq 1$.

Prob. 4 Show that the absolute continuity of the non-negative function f on the finite interval $[a, b]$ does not imply that f^p is absolutely continuous if $0 < p < 1$.
(b) Show that if f is non-negative and absolutely continuous on $[a, b]$ then so is f^p, $p \geq 1$.