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Prob. 1

Let $E = F = [0,1]$, $\Sigma_1 = \Sigma_2 = \mathcal{B}([0,1])$, $\mu = m$ and $\nu = \mu_c$ on $[0,1]$. Consider

$$V = \{(x,y) : x = y, (x,y) \in E \times F\}.$$

Prove that V is measurable and find the integrals $\int_F \int_E \chi_V d\mu d\nu$ and $\int_E \int_F \chi_V d\nu d\mu$.

Prob. 2

Let $\{f_n\}$ be a sequence of integrable functions such that $\sum_{n=1}^{\infty} \int |f_n| \, d\mu < \infty$. Prove that $\sum_{n=1}^{\infty} f_n$ converges a.e., its sum f is integrable and that $\int f d\mu = \sum_{n=1}^{\infty} \int f_n d\mu$.

Prob. 3

Show that

$$\lim_{n\to\infty} \int_0^1 \frac{n^p x^r \log x dx}{1 + n^2 x^2} = 0$$

where $r > 0$, $0 < p < \min\{2, 1 + r\}$.

Prob. 4 Show that

$$\lim_{b\to 1^-} \int_0^b \sum_{n=1}^{\infty} \frac{x^{n-1} dx}{\sqrt{n}} = \sum_{n=1}^{\infty} n^{-3/2}.$$

Prob. 5

Prove that $\int_A f d\mu = 0 \Leftrightarrow f = 0 \; \mu - a.e.$ on $A \in \Sigma$.