King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

April 25, 2013, Semester 122
Math 513.02- Test on Chapter 9

Name: _____ I.D. # _____

1. Find the eigenvalues and eigenfunctions of $y''+\lambda y=0;\ y'(0)=y'(\pi)=0$.
2. Find the first two coefficients in the Legendre polynomial expansion for $f(x)=\begin{cases}-1 & -1<x<0\\ x & 0<x<1\end{cases}$.
3. Chebyshev polynomials of type I ($H_n(x),\ n=0,1,2,...$) satisfy the differential equation $(1-x^2)y''-xy'+n^2y=0$ on $[-1,1]$. One can show that these polynomials are orthogonal with respect to a weight $w(x)$ (i.e. $\int_{-1}^{1} w(x)H_n(x)H_m(x)=0 \quad if \quad n\neq m$). Find $w(x)$.
4. If $\delta(x-a)=\sum_{k=1}^{\infty}A_k J_1(\mu_k x);\ 0\le x,a<1$, where $\delta(\bullet)$ is Dirac delta function, $J_1(\mu_k x)$ is Bessel function, and μ_k is the kth positive root of $J_1(\mu_k)=0$. Find A_k.