King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

March 20, 2013, Semester 122
Math 513.02- First Examination

Name: ____ I.D. # ________________________

1. Given $f(x)=\begin{cases} 0 & if \ \ -\pi < x < 0 \\ 1 & if \ 0 \le x < \pi \end{cases}$

a) Find the Fourier series of $f(x)$ on the given interval.

b) Write the series in part (a) in the cosine phase angle form.

c) Evaluate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$.

2. Use Laplace transform to solve the initial value problem $y'' + y = f(t);\ y(0) = y'(0) = 0$, where $f(t) = \begin{cases} \cos t & 0 \le t < \pi \\ 0 & t \ge \pi \end{cases}$.

3. Find $\mathcal{L}^{-1}\left[\frac{1}{s^2 - 2s - 2}\right]$.

4. Find $\mathcal{L}\left[\ t \int_0^t x e^{-x}\, dx\right]$.

5. Find the Fourier transform of $f(t) = 7e^{-|3t-2|} \sin t$.

6. Use Fourier transform to solve $y^{(4)} - y = 4\delta(t)$.

IMPORTANT:

Each problem must start on a new page.
Write clearly and neatly.
Calculators are not allowed.

Q1: 7 points
Q2: 7 points
Q3: 3 points
Q4: 3 points
Q5: 3 points
Q6: 7 points

Total: 30 points

Formulas

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$$

$$\mathcal{L}\{t^n\} = n!/s^{n+1}$$

$$\mathcal{L}\{\sin kt\} = k(s^2 + k^2)^{-1}$$

$$\mathcal{L}\{\cos kt\} = s(s^2 + k^2)^{-1}$$

$$\mathcal{L}\{\sinh kt\} = k(s^2 - k^2)^{-1}$$

$$\mathcal{L}\{\cosh kt\} = s(s^2 - k^2)^{-1}$$

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

$$\mathcal{L}\{f(t-a)H(t-a)\} = e^{-as}F(s)$$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$f * g = \int_0^t f(x)g(t-x)\,dx$$

Fourier Series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}\right]$$

$$a_n = \frac{1}{L}\int_r^{r+2L} f(t)\cos\frac{n\pi t}{L}\,dt$$

Integrals

$$\int \cos(a\,t)\cos(b\,t)\,dt = \frac{\sin[(a-b)\,t]}{2(a-b)} + \frac{\sin[(a+b)\,t]}{2(a+b)}$$

$$\int \sin(a\,t)\cos(b\,t)\,dt = -\frac{\cos[(a-b)\,t]}{2(a-b)} - \frac{\cos[(a+b)\,t]}{2(a+b)}$$

$$\int \sin(a\,t)\sin(b\,t)\,dt = \frac{\sin[(a-b)\,t]}{2(a-b)} - \frac{\sin[(a+b)\,t]}{2(a+b)}$$

Fourier Transform

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\,e^{-i\omega t}dt = F(\omega)$$

$$\mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\,e^{i\omega t}d\omega = f(t)$$

$$\mathcal{F}\{t^n\,H(t)\,e^{-at}\} = n!/(a+i\,\omega)^{n+1}, \mathrm{Re}(a) > 0, n = 1,2,..$$

$$\mathcal{F}\{e^{-a|t|}\} = \frac{2a}{\omega^2 + a^2},\ a > 0$$

$$\mathcal{F}\{e^{i\omega_0 t}\} = 2\pi\delta(\omega - \omega_0)$$

$$\mathcal{F}\{\delta(t)\} = 1$$

$$\mathcal{F}\{\mathrm{sgn}(t)\} = \begin{cases} 2/(i\omega), & \omega \neq 0 \\ 0, & \omega = 0 \end{cases}$$

$$\mathcal{F}\{H(t)\} = \pi\delta(\omega) + \frac{1}{i\omega}$$

$$\mathcal{F}\{f(t-\tau)\} = e^{-i\omega\tau}F(\omega)$$

$$\mathcal{F}\{f(t)e^{i\omega_0 t}\} = F(\omega - \omega_0)$$

$$\mathcal{F}\{f^{\,n}(x)\} = (i\omega)^n F(\omega)$$

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(x)\,g(t-x)\,dx$$

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$