King Fahd University of Petroleum & Minerals ID #
Department of Mathematics & Statistics
MATH 455/ Exam II/ 122 Name

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1. [10pts] (a) Verify that $3^8 \equiv -1 \pmod{17}$ and deduce that 3 is a primitive root mod 17.
(b) Let p be prime. Show that if a is an integer with order $4 \bmod p$, then $p \equiv 1 \pmod 4$.

2. [15pts] Let p be an odd prime.

(a) Give in terms of p the value of $\left(\frac{-2}{p}\right)$

(b) Prove that if a and b are integers such that $a \equiv b (\mathrm{mod}\, p)$ then $\left(\frac{a}{p}\right) = \left(\frac{b}{p}\right)$

(c) State the quadratic reciprocity law and compute $\left(\frac{1499}{2999}\right)$ (note that 1499 and 2999 are primes).

3. [15pts] (a) Let x, y be real numbers. Prove that $[x] + [y] \leq [x + y]$ and that $[x - y] \leq [x] - [y]$
(b) For what real numbers x do we have $[x + 1/2] + [x - 1/2] = [2x]$? Justify.
(c) Find the highest power of 9 that divides $(3^8)!$

4. [10pts] For $n \in \mathbb{N}$ let $d(n)$ and $\sigma(n)$ denote respectively the number and the sum of the positive divisors of n.

(a) Find all primes p such that $d(p^2) + \sigma(p^2) + \phi(p^2) = 3p^2$.

(b) Prove that the function h given by $h(n) = \sigma(n)\phi(n)$ is multiplicative and that $h(n) \leq n^2$ for all n in $\mathbb{N}$