King Fahd University of Petroleum & Minerals ID #
Department of Mathematics & Statistics
MATH 455/ Exam I/ 122 Name

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1. [10pts] (a) Let $d = (455, 196)$. Find d and find integers x and y such that $455x + 196y = d$
(b) Show that there are infinitely many integers m, n such that $4m - 55n = 1$

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2. [15pts] (a) Show that if a, b are coprime integers then $(2ab, a + b) \leq 2$.
(b) Show that if a, b, c are positive integers and p is a prime such that $[a, b] = p^c(a, b)$ then either $a \mid b$ or $b \mid a$
(c) Give an example of positive integers a, b, m with $[a, b] = m(a, b)$ but such that $a \nmid b$ and $b \nmid a$.

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3. [10pts] Let m be a positive integer and a be an integer such that $(a, m) = 1$
(a) Prove that the set $\{a + 1, 2a + 1, \ldots, ma + 1\}$ is a complete residue system $\bmod m$
(b) If $\{r_1, r_2, \ldots, r_k\}$ is a reduced residue system $\bmod m$, is it true that $\{r_1a + 1, r_2a + 1, \ldots, r_ka + 1\}$ is also a reduced residue system $\bmod m$? Either prove or give a counterexample.

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4. [15pts] (a) Solve the system of congruences

$$3x \equiv 2 \pmod 5$$
$$2x \equiv 1 \pmod 7$$
$$4x \equiv 5 \pmod{11}$$

(b) Solve the congruence $x^3 + x + 2 \equiv 0 \pmod{25}$
(c) Solve the congruence $x^{12} + x^{11} + x^{10} \equiv 2 \pmod{11}$