KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 302: FINAL EXAM, SEMESTER (122), MAY 20, 2013

Time: 08:00 to 11:00 am

Name : ....

ID : .... Section : ....

Exercise	Points
1	___ / 20
2	___ / 20
3	___ / 20
4	___ / 20
5	___ / 20
6	___ / 20
7	___ / 20
Total	___ / 140

Exercise 1. Let $A = \begin{pmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{pmatrix}$.

(1) Show that $\det(A - \lambda I) = -(\lambda - 1)^2(\lambda + 3)$.

(2) Is A diagonalizable? If yes, construct a matrix P that diagonalizes A.

Exercise 2. Use Divergence theorem to evaluate the flux $\displaystyle\iint_S F.\mathbf{n}\ \mathrm{d}S$ of

$$F(x,y,z) = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$$

across the surface S given by $z = 4 - x^2 - y^2$ for $z \geq 3$.

Exercise 3. Let $f(z) = Re(z)|z|^2$. We denote by $x = \text{Re}(z)$, $y = \text{Im}(z)$, $u(x, y) = \text{Re}(f(z))$ and $v(x, y) = \text{Im}(f(z))$.

(1) Find all (x, y) at which u, v satisfy Cauchy-Riemann Equations.
(2) Show that $f(z)$ is differentiable at $z_0 = 0$.

Exercise 4. Find all complex numbers z such that $\sin z = 2$.

Exercise 5. Use Cauchy Integral Formula to evaluate the integral

$$\oint_C \frac{e^z \sin z}{(z-2)^3}\, \mathrm{d}z,$$

where C is the positively oriented circle $|z| = 3$.

Exercise 6. Let $f(z) = ze^{\frac{2}{z^2}}$

(1) Find the Laurent series of the function $f(z)$ about $z_0 = 0$ in the region $0 < |z|$.

(2) Use (1) to evaluate the contour integral

$$I = \oint_{\mathcal{C}} f(z)\,\mathrm{d}z,$$

where $\mathcal{C}$ is the positively oriented circle $|z| = 1$.

Exercise 7. Use the residue theorem to evaluate

(1) $\oint\limits_{\mathcal{C}} \cot(z)\,\mathrm{d}z$
where $\mathcal{C}$ is the positively oriented circle $|z| = 1$.

(2) $\oint\limits_{\mathcal{C}} \frac{e^z}{z(z-2)^3}\,\mathrm{d}z$
where $\mathcal{C}$ is the positively oriented circle $|z| = 3$.