KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 302: EXAM II, SEMESTER (122), APRIL 06, 2013

Time: 08 : 00 to 10: 00 pm

Name : .....

ID : ..... Section : .....

Exercise	**Points**
1	_____ 10
2	_____ 20
3	_____ 20
4	_____
5	_____
6	_____
Total	_____ 100

Exercise 1.

(a) Let $\mathcal{C}$ be the curve with position vector $\mathbf{r}(t) = (3\cos(t^2), 3\sin(t^2))$, $0 \leq t \leq \frac{\pi}{2}$. Evaluate the length $l(\mathcal{C})$ of the curve $\mathcal{C}$.

(*b*) Given the surface S defined by: $z = 2x^2 + y^2 - 1$. Find an equation of the tangent plane to the surface S at $(1, 1, 2)$.

Exercise 2. Let $F(x,y) = (e^{-x^2}\sin x + y^3)\mathbf{i} + (x^2y^2 + e^{-y^2}\sin y)\mathbf{j}$ be a force moving a particle from the point $P = (1,-1)$ to the point $Q = (1,-1)$ along the positively oriented simple closed path $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2 \cup \mathcal{C}_3 \cup \mathcal{C}_4 \cup \mathcal{C}_5$, where

- $\mathcal{C}_1$ is the the line segment joining $(1,-1)$ and $(1,1)$,
- $\mathcal{C}_2$ is the line segment joining $(1,1)$ and the origin $(0,0)$,
- $\mathcal{C}_3$ is the line segment joining $(0,0)$ and $(-1,1)$,
- $\mathcal{C}_4$ is the line segment joining $(-1,1)$ and $(-1,-1)$,
- $\mathcal{C}_5$ is the line segment joining $(-1,-1)$ and $(1,-1)$.

(1) Graph the path $\mathcal{C}$ and precise the direction of the motion.
(2) Using Green's theorem, evaluate the work done by the force F in moving the particle.

Exercise 3. Let $\mathcal{C}$ be the curve obtained by intersecting the plane $z = x$ and the cylinder $x^2 + y^2 = 1$, oriented counter-clockwise when viewed from above. Let S be the portion of the plane that lies inside the cylinder, oriented with the upward-pointing normal. For $F(x, y, z) = x\mathbf{i}+z\mathbf{j}+2y\mathbf{k}$, verify Stokes' theorem by computing both

$$\oint_{\mathcal{C}} F.dR \qquad \text{and} \qquad \int\int_{S} (Curl(F).\mathbf{n})dS.$$

Exercise 4.

Exercise 5.

Exercise 6.