KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 302: EXAM I, SEMESTER (122), FEBRUARY 26, 2013

Time: 08 : 00 to 10: 00 pm

Name: ....

I.D: .... Section: ....

Exercise	Points
1	____ / 25
2	____ / 25
3	____ / 25
4	____ / 25
Total	____ / 100

Exercise 1.

(1) Let m be a real number and

$$S_m = \{(x, y) \in \mathbb{R}^2 \mid m(x-1) + m^2(y-2) = -m^2 - 3m + 1\}.$$

Find the value(s) of m such that S_m is a subspace of $\mathbb{R}^2$.

(2) Find a basis and the dimension of the following subspace of $\mathbb{R}^3$:

$$S = \{(x, y, z) \mid x - y + 5z = 0\}.$$

Exercise 2. Let k be a real number. Consider the nonhomogeneous system $AX = B$, where

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 2 & 0 \\ 3 & 4 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 2 \\ k \end{pmatrix} \text{ and } X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

(a) Find rank(A) and rank$([A \vdots B])$.

(b) Use (a) to find the value of k for which the system $AX = B$ has infinitely many solutions.

(c) Is there a value of k for which $AX = B$ has a unique solution?

Exercise 3. Let

$$A = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ -1 & 1 & 2 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$

(i) Use Gauss-Jordan Method to find the inverse of A.

(ii) Use the matrix A^{-1} to solve the following system

$$\begin{cases} x_1 \phantom{{}+x_2} - x_3 = 1 \\ x_2 + x_3 = 2 \\ -x_1 + x_2 + 2x_3 + x_4 = 3 \\ + x_3 - x_4 = 4 \end{cases}$$

Exercise 4. Let

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

(a) Find a nonsingular matrix P and a diagonal matrix D such that

$$P^{-1}AP = D.$$

(b) Determine whether P is an orthogonal matrix.