MATH 280-01 (122)
HW # 4

Q1. Find the dimension and basis of the solution space W of the system of linear equations:

$$\begin{aligned} x + 2y - 4z + 3r - s &= 0 \\ x + 2y - 2z + 2r + s &= 0 \\ 2x + 4y - 2z + 3r + 4s &= 0 \end{aligned}$$

Q2. Let $u = [x_1\ x_2]^T$, $v = [y_1\ y_2]^T$ belong to $\Re^2$?

(a) verify that the following is an inner product on $\Re^2$:
$(u, v) = x_1y_2 - 2x_1y_2 - 2x_2y_1 + 5x_2y_2$

(b) for what values of k is the following:
$(u, v) = x_1y_1 - 3x_1y_2 - 3x_2y_1 + kx_2y_2$
an inner product on $\Re^2$

Q3. Let $u_1 = 1$, $u_2 = x$, $u_3 = x^2$, and $u_4 = x^3$ then $W = \{u_1, u_2, u_3, u_4\}$ is a basis for P_3, all polynomials with degree less than or equal 3, with inner product:
$(p(x), q(x)) = \int_0^1 p(x)q(x)dx$
Find an **orthogonal** basis for P_3.

Q4. Let $S = \{[1\ 2\ -2]^T, [4\ 3\ 2]^T, [1\ 2\ 1]^T\}$ be a basis for $\Re^3$, use the Gram-Schmidt process to obtain an orthonormal basis.

Q5.Given the basis $\{u_1, u_2, u_3\}$ for $\Re^3$ with

$$u_1 = \begin{bmatrix} \frac{1}{3\sqrt{2}} \\ \frac{1}{3\sqrt{2}} \\ \frac{-4}{3\sqrt{2}} \end{bmatrix}, u_2 = \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}, u_3 \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \\ 0 \end{bmatrix}$$

a)Show that $T = \{u_1, u_2, u_3\}$ is an orthonormal basis for $\Re^3$

b) Let

$$X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Write X as a linear combination of $u_1, u_2,$ and u_3 using Theorem 3.5 and use $[X]_T$ to compute $\|X\|$