King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 280-01(Term 122)
Final Exam

NAME: ....

ID #: . . . .

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Question	Score
1 (14 pts)	
2 (12 pts)	
3 (12 pts)	
4 (12 pts)	
5 (12 pts)	
6 (12 pts)	
7 (12 pts)	
8 (14 pts)	
9 (12 pts)	
10 (12 pts)	
11 (12 pts)	
12 (14 pts)	
Total (150)	

Q1. Let A be an $n \times n$ nonsingular matrix and $adj(A)$ its classical adjoint.

(a) What is the relation between A^{-1} and $adj(A)$

(b) Show that if $adj(A)$ is symmetric, then A is symmetric.

Q2.Writer the real valued function $g(\mathbf{x}) = 2x_1^2 + 2x_2^2 + 10x_1x_2$ in the quadratic form $g(\mathbf{x}) = \mathbf{x^T}A\mathbf{x}$ for some *symmetric* matrix A.

Use the Principle Axes Theorem to find an equivalent quadratic form $h(\mathbf{y})$ where $\mathbf{y} = (y_1\ y_2)^T$

Q3. let

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

(a) Is A diagonalizable? Why?

(b) Find the eigenvalues and the associated eigenvectors of A.

Q4.Let $L : \Re^4 \to \Re^3$ be a linear transformation with

$$\begin{aligned} L(e_1) &= e_2 - 3e_3 \\ L(e_2) &= 2e_1 - 3e_2 + 10e_3 \\ L(e_3) &= 3e_1 + 4e_2 - 6e_3 \\ L(e_4) &= 4e_1 + 5e_2 - 7e_3 \end{aligned}$$

where $\{e_1, e_2, e_3, e_4\}$ is the standard basis for $\Re^4$.

(a)Find a matrix representation of L

(b)What is the rank of L?

Q5.

(a) Write the definition of similar matrices.

(b) Show that if A and B are invertible matrices, then AB and BA are similar.

Q6. If $\mathbf{x}$ is an eigenvector of A corresponding to λ, what is $A^3\mathbf{x}$?

Q7. Let A be an $n \times n$ matrix such that $A = PDP^{-1}$ where D is a diagonal matrix. Show that $A^k = PD^kP^{-1}$. What are the diagonal entries of D^k compared to those of D?

Q8. Let x_1, x_2 and x_3 be linearly independent vectors in $\Re^4$ and let A be a nonsingular 4×4 matrix. Prove that if $y_1 = Ax_1,\ y_2 = Ax_2$, and $y_3 = Ax_3$ the y_1, y_2, and y_3 are linearly independent.

Q9. Let

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix}$$

(a) Find the cofactors A_{11}, A_{21} and A_{31}

(b) Use part (a) to find the determinant of A.

Q10. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 5 & 5 \\ 0 & 0 & 3 & 3 \end{bmatrix}$$

(a) Find a basis for the null space of A (Solution space of $Ax = 0$).

(b)What is the nullity of A?

Q11. Let

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 3 & 4 \\ 7 & -5 & 2 \end{bmatrix}$$

(a) Determine whether or not the **rows** of A are orthogonal.

(b) Determine whether or not A is an orthogonal matrix and why.

Q12. **TRUE** or **FALSE**

a) (   ) For any matrix A, the product AA^T is diagonalizable.

b) (   ) If A and B are symmetric matrices, then AB is symmetric.

c) (   ) If A is an invertible 3×3 matrix, then its rows form a basis of $\Re^3$.

d) (   ) If A is an $m \times n$ matrix and $B = rref(A)$, then $ker(A) = ker(B)$.

e) (   ) If A and B are matrices whose eigenvalues are the same, then A and B are similar.

f) (   ) Suppose $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors and $(\mathbf{u}, \mathbf{v}) = 0$, then $\mathbf{u}$ and $\mathbf{v}$ are linearly dependent.

g) (   )

$$A = \begin{bmatrix} 1 & 5 \\ -1 & -3 \end{bmatrix}$$

has no eigenvalues