

King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 280-01(Term 122)
Exam II

NAME:

ID #:

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Question	Score
1 (10 pts)	
2 (12 pts)	
3 (12 pts)	
4 (10 pts)	
5 (14 pts)	
6 (10 pts)	
7 (12 pts)	
8 (12 pts)	
9 (8 pts)	
Total (100)	

Q1. Let $L : \mathfrak{R}^2 \rightarrow \mathfrak{R}^2$ be a linear transformation and let A be the standard matrix representation of L . Define L^2 by $L^2(x) = L(L(x))$ for all $x \in \mathfrak{R}^2$. Show that L^2 is a linear transformation and its matrix representation is A^2

Q2. Let $L : P_3 \rightarrow P_3$ be a linear transformation defined by

$$L(p(x)) = xp'(x)$$

Find the kernel and range of L .

Q3. let

$$S = \left\{ u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, u_3 = \begin{bmatrix} 5 \\ -4 \\ -1 \end{bmatrix} \right\}$$

- (a) Show that S is orthogonal and S is a basis for \mathfrak{R}^3
(b) Write $(1, 5, -7)^T$ as a linear combination of u_1, u_2, u_3 .

Q4. Find k so that $f(t) = t + k$ and $g(t) = t^2$ are orthogonal with $(f, g) = \int_0^1 f(t)g(t)dt$

Q5. Find an **orthonormal** basis for the subspace U of \mathfrak{R}_4 spanned by the vectors:

$$v_1 = (1, 1, 1, 1), \quad v_2 = (1, 1, 2, 4), \quad v_3 = (1, 2, -4, -3)$$

Q6. let w_1 and w_2 be two nonzero **orthogonal** vectors in V and let v be any vector in V .
If $u = v - c_1w_1 - c_2w_2$ is **orthogonal** to w_1 and w_2 then find c_1 and c_2

Q7. Let $L : \mathfrak{R}^2 \rightarrow \mathfrak{R}^2$ be a linear transformation. Let $(1, 2)^T, (0, 1)^T$ be a basis for \mathfrak{R}^2 . Suppose $L(1, 2)^T = (2, 3)^T$ and $L(0, 1)^T = (1, 4)^T$, find a formula for $L(a, b)^T$

Q8. Let u and v be any two vectors in an inner product space V .

(a) state the **Cauchy-Schwarz** inequality.

(b) state the **Triangle** inequality

Q9. TRUE or FALSE

a) () Let $L : \mathfrak{R}^n \rightarrow \mathfrak{R}^2$ be a linear transformation. If $L(x_1) = L(x_2)$, then the vectors x_1 and x_2 must be equal

b) () If $L : V \rightarrow V$ is a linear transformation and $x \in \text{Ker}L$ then $L(v+x) = L(v)$ for all $v \in V$

c) () Let $L : \mathfrak{R}^n \rightarrow \mathfrak{R}^n$ be a linear transformation. If A is the standard matrix representation of L , then an $n \times n$ matrix B will also be a matrix representation of L if and only if B is similar to A

d) () Let A, B , and C be $n \times n$ matrices. If A is similar to B and B is similar to C , then A is similar to C .