King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 280-01(Term 122)
Exam I

NAME: ....

ID #: . . . .

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Q1. Using the Gauss-Jordan reduction method find all solutions of the following homogeneous system:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ 4x_1 - x_2 + 3x_3 + x_4 &= 0 \\ 3x_1 + 5x_3 + x_4 &= 0 \end{aligned}$$

Q2. determine if the following vectors are linearly independent:

a)

$$\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -8 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 2 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Q3. let

$$S = \{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 11 \\ 10 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 6 \\ 4 \end{bmatrix} \}$$

Let $W = \text{Span } S$, where W is a subspace of $\Re^3$.
Find a subset of S that is a basis for W

Q4. Let A be n x n nonsingular matrix. Show that $Ax = b$ has a solution for all b and show that the solution is unique.

Q5. Let S be a subset of $\Re^3$ such that

$$S = \{(a\ b\ c)^T \in \Re^3 \mid c = 2a + b\}$$

Show that S is a subspace of $\Re^3$

Q6. let

$$S = \{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} \}$$

be an ordered basis for $\Re^3$. Let $v \in \Re^3$ such that $[v]_S = [2 \ -4 \ 3]^T$. Find v

Q7. Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -3 & 2 \\ -1 & -2 & 1 \end{bmatrix}$$

Q8. Determine if the vectors $[2\ 2\ 2]^T, [0\ 0\ 3]^T, [0\ 1\ 1]^T$ form a basis for $\Re^3$

Q9. Let V and W be vector spaces and let L be the linear transformation $L : V \to W$. State and **explain** all the properties that must be satisfied in order for L to be an isomorphism of V onto W

Q10. **TRUE** or **FALSE**
(for **False** statement give a counter example)

a) ( ) $AB = BA$ for all square matrices A and B of the same size.

b) ( ) Any collection of n linearly independent vectors in $\Re^n$ form a basis for $\Re^n$.

c) ( ) The set of all n x n nonsingular matrices is closed under addition.

d) (   ) $\Re^2$ is a subspace of $\Re^3$.

e) (   ) For any 2 x 2 matrices A and B we have $(A+B)^2 = A^2 + 2AB + B^2$

f) (   ) Any n vectors which span a vector space V of dimension n form a basis for V.