King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
(Math 260)

Second Exam
Term 122
Monday, April 15, 2013
Net Time Allowed: 135 minutes

Key

Name:	
ID:	
Section No:	

(Show all your steps and work)

Q		Points
1		12
2		12
3		12
4		10
5		10
6		10
7		10
8		12
9		12
Total		**100**

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(1) Given the matrix $A=\begin{bmatrix}4&3&2\\5&6&3\\3&5&2\end{bmatrix}$ [12 points]

6 pts for steps

(a) Use Elementary Row Operations to find A^{-1}

2

$$\left[\begin{array}{ccc|ccc}4&3&2&1&0&0\\5&6&3&0&1&0\\3&5&2&0&0&1\end{array}\right] \overset{-R_3+R_1}{\sim} \left[\begin{array}{ccc|ccc}1&-2&0&1&0&-1\\5&6&3&0&1&0\\3&5&2&0&0&1\end{array}\right]$$

$$\overset{-R_3+R_2}{\sim} \left[\begin{array}{ccc|ccc}1&-2&0&1&0&-1\\2&1&1&0&1&-1\\3&5&2&0&0&1\end{array}\right] \overset{-2R_1+R_2}{\sim} \left[\begin{array}{ccc|ccc}1&-2&0&1&0&-1\\0&5&1&-2&1&1\\3&5&2&0&0&1\end{array}\right]$$

$$\overset{-3R_1+R_3}{\sim} \left[\begin{array}{ccc|ccc}1&-2&0&1&0&-1\\0&5&1&-2&1&1\\0&11&2&-3&0&4\end{array}\right] \overset{-2R_2+R_3}{\sim} \left[\begin{array}{ccc|ccc}1&-2&0&1&0&-1\\0&5&1&-2&1&1\\0&1&0&1&-2&2\end{array}\right]$$

$$\overset{R_2\leftrightarrow R_3}{\sim} \left[\begin{array}{ccc|ccc}1&-2&0&1&0&-1\\0&1&0&1&-2&2\\0&5&1&-2&1&1\end{array}\right] \overset{-2R_2+R_1}{\sim} \left[\begin{array}{ccc|ccc}1&0&0&3&-4&3\\0&1&0&1&-2&2\\0&5&1&-2&1&1\end{array}\right]$$

Finally, $-5R_2+R_3$ $\left[\begin{array}{ccc|ccc}1&0&0&3&-4&3\\0&1&0&1&-2&2\\0&0&1&-7&11&-9\end{array}\right]$ Done!

(b) Use **part (a)**, to find a 3x4 matrix X such that

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$$\begin{bmatrix}4&3&2\\5&6&3\\3&5&2\end{bmatrix}X=\begin{bmatrix}3&-1&2&6\\7&4&1&5\\5&2&4&1\end{bmatrix}$$

$$X = A^{-1}\begin{bmatrix}3&-1&2&6\\7&4&1&5\\5&2&4&1\end{bmatrix} = \begin{bmatrix}-4&-13&14&1\\-1&-5&8&-2\\11&33&-39&4\end{bmatrix}$$

2

2

(2) [12 points]

a) Use **cofactor expansion** to evaluate the determinant of $\begin{vmatrix} 2 & 0 & 0 & -3 \\ 0 & 1 & 11 & 12 \\ 0 & 0 & 5 & 13 \\ -4 & 0 & 0 & 7 \end{vmatrix}$

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(Expand along the row or column that minimize the amount of computation)

pick 1st Row $[2 \ 0 \ 0 \ -3]$ — 2

$$\Rightarrow 2\begin{vmatrix} 1 & 11 & 12 \\ 0 & 5 & 13 \\ 0 & 0 & 7 \end{vmatrix} - (-3)\begin{vmatrix} 0 & 1 & 11 \\ 0 & 0 & 5 \\ -4 & 0 & 0 \end{vmatrix} =$$

$$2 \cdot 1\begin{vmatrix} 5 & 13 \\ 0 & 7 \end{vmatrix} + 3(-4)\begin{vmatrix} 1 & 11 \\ 0 & 5 \end{vmatrix} =$$

$$2(35-0) - 12(5-0) = 70 - 60 = \boxed{10}$$

6-points for steps

b) The square matrix A is called **ORTHOGONAL** provided that $A^T = A^{-1}$. Show that the determinant of such a matrix must be either $+1$ or -1.

4

$$A^T = A^{-1} \Rightarrow \det(A^T) \overset{1}{=} \det(A^{-1}) \Rightarrow$$

$\det(A^T) = \det(A)$ 1 & $\det(A^{-1}) = \frac{1}{\det(A)}$ 1 so

$$\det(A) = \frac{1}{\det(A)} \Rightarrow \det(A) = \pm 1$$

1

(3) Let $u=(2,1,0), v=(1,2,1)$ and $w=(0,1,2)$ be vectors in R^3. [12 points]

a) Show that the vectors: u, v and w are linearly independent.

$$\begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = 6-2 = 4 \neq 0$$

6 pts

or by any other Methods!

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 7 \\ 1 & 2 & 1 & 7 \\ 0 & 1 & 2 & 7 \end{array}\right] \underset{\sim}{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 2 & 1 & 0 & 7 \\ 0 & 1 & 2 & 7 \end{array}\right] \underset{\sim}{-2R_1+R_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & -3 & -2 & 7 \\ 0 & 1 & 2 & 7 \end{array}\right]$$

$$\underset{\sim}{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & 2 & 7 \\ 0 & -3 & -2 & -7 \end{array}\right] \underset{\sim}{3R_2+R_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & -7 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 4 & 14 \end{array}\right]$$

so, by Back sub. $\boxed{c = 7/2} \Rightarrow \boxed{b = 0} \Rightarrow \boxed{a = 7/2}$

b) Express the vector $y=(7,7,7)$ as a linear combination of the vectors: u, v and w

2

Set $y = au + bv + cw$, then

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \\ 7 \end{bmatrix}$$

Finding a, b and c 4

(4) Let W be a non-empty subset of the vector space V. [10 points]

a) If $au + bv \in W$ for any two vectors $u, v \in W$ and any scalars a,b, then show that W is a subspace of V.

3 choose $a = b = 1 \Rightarrow u + v \in W, \forall u, v \in W$

3 choose $b = 0 \Rightarrow au \in W, \forall a \in \mathbb{R}$
$\forall u \in W$

So, W is a Supspace of V

6 pts

b) If W is subspace of V, then show that $o \in W$.

Again, let $u \in W \Rightarrow -u \in W$
2

So $u + (-u) = 0 \in W$

2

4pts

(5) Find a **BASIS** for the solution space of the homogeneous system [10 points]

$$x_1 - 3x_2 - 9x_3 - 5x_4 = 0$$
$$2x_1 + x_2 - 4x_3 + 11x_4 = 0$$
$$x_1 + 3x_2 + 3x_3 + 13x_4 = 0$$

The System is Matrix form

$$\begin{bmatrix} 1 & -3 & -9 & -5 \\ 2 & 1 & -4 & 11 \\ 1 & 3 & 3 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Apply row-operations on the Matrix

$$\begin{bmatrix} 1 & -3 & -9 & -5 \\ 2 & 1 & -4 & 11 \\ 1 & 3 & 3 & 13 \end{bmatrix} \sim\sim \begin{bmatrix} 1 & 0 & -3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ (4)

So, 2-free variables set $x_4 = t$ and $x_3 = s$

$$\Rightarrow x_1 = \overset{3s-4t}{\cancel{s-2t}} \text{ and } x_2 = \overset{-2s-3t}{\cancel{-3s+t}}$$ (2)

$$X = (x_1, x_2, x_3, x_4) = (\overset{3s-4t}{\cancel{s-2t}}, \overset{-2s-3t}{\cancel{-3s+t}}, s, t)$$ (2)

$$= s(\overset{3,-2,1,0}{\cancel{1,-3,1,0}}) + t(\overset{-4,-3,0,1}{\cancel{-2,1,0,1}})$$ (2)

$$\mathcal{B} = \{(\overset{3,-2,1,0}{\cancel{1,-3,1,0}}), (\overset{-4,-3,0,1}{\cancel{-2,1,0,1}})\} - \boxed{\text{2-Dim}}$$

6) If $y = e^{3x}$ is one solution of $y''' + 3y'' - 54y = 0$,.

a) Find the other two solutions. [10 points]

The charac. eq: $m^3 + 3m^2 - 54 = 0$ 2

Since e^{3x} is a Sol $\Longrightarrow$ $(m-3)$ is factor 1

perform $\dfrac{m^3 + 3m^2 - 54}{m-3} = m^2 + 6m + 18$

$= m^2 + 6m + 9 + 9$

2 $= (m+3)^2 + 9$

$(m+3)^2 + 9 = 0 \Longrightarrow m = -3 \pm 3i$ 1

So $y_2 = e^{-3x}\cos 3x$ 1 and $y_3 = e^{-3x}\sin 3x$ 1

where $y_1 = e^{3x}$

b) Find the general solution.

$y_{general} = c_1 y_1 + c_2 y_2 + c_3 y_3$ 2

7) Determine the general solution to $(D-3)\left(D^2+2D+2\right)^2 y=0$ [10 points]

(Where D is the differential operator)

charach. poly: $(m-3)(m^2+2m+1+1)^2$ 2

$m=3$ and $\left((m+1)^2+1\right)^2=0$

2 $\Longrightarrow$ $m=-1\pm i$ (multip #2)

5-Sol: $y_1=e^{3x}$ 1

$y_2=e^{-x}\cos x$

$y_3=e^{-x}\sin x$ 1

$y_4=xe^{-x}\cos x$

$y_5=xe^{-x}\sin x$ 2

$y_{gen}=c_1y_1+c_2y_2+c_3y_3+c_4y_4+c_5y_5$ 2

8) Determine the appropriate form for a particular solution of the fifth-order Differential Equation: $(D-1)^3(D^2-4)y = xe^x + e^{2x} + e^{-2x}$ [12 points]

(Do not evaluate the values of the constants).

$*\ (D-1)^3 = 0 \Longrightarrow$ repeated 3-times !

$c_1 + c_2 x + c_3 x^2$ 3

$D^2 - 4 = 0 \Longrightarrow \pm 2 \Longrightarrow c_4 e^{2x} + c_5 e^{-2x}$ 2

So $y_c = (c_1 + c_2 x + c_3 x^2) e^x + c_4 e^{2x} + c_5 e^{-2x}$ 1

By Considering the R-H-S: $xe^x + e^{2x} + e^{-2x}$

The Best proposed particular is

$(A + Bx) e^x + C e^{2x} + D e^{-2x}$. +3

Then $y_p = ((A + Bx) e^x) x^3 + (C e^{2x}) x$
$+ (D e^{-2x}) x$.

+3

9) By using the **variation of parameters method**, find the particular and general solutions of $y'' + 9y = 2\sec 3x$.. [12 points]

$m^2 + 9 = 0 \Rightarrow m = \pm 3i \Rightarrow$ $y_1 = \cos 3x$, $y_2 = \sin 3x$ (2)

$\boxed{y_c = c_1 y_1 + c_2 y_2}$

$y_1' = -3\sin 3x$ and $y_2' = 3\cos 3x$

$$W = \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix} = 3$$ (2)

$u_1' = -\frac{2}{3} \sin 3x \cdot \sec 3x = -\frac{2}{3} \tan 3x$

$u_2' = \frac{2}{3} \cos 3x \cdot \sec 3x = 2/3$ (2)

$\Rightarrow u_1 = -2/3 \int \tan 3x \, dx$ & $u_2 = \int 2/3 \, dx$

$u_1 = \frac{2}{9} \ln|\cos 3x|$ & $u_2 = 2/3 x$ (2)

so $y_p = u_1 y_1 + u_2 y_2$ (2)

and $y_{general} = y_c + y_p$ (2)

$y_p = \frac{2}{9}(3x \sin 3x + \cos 3x \ln|\cos 3x|)$