Math 102

Quiz#3

Key "I"

40

Name:

ID No.:

Serial No.:

1

1. Evaluate $I = \int_0^9 \frac{2\log_{10}(s+1)}{s+1}\, ds$.

Q 43 page 426

$$I = \int_0^9 \frac{2\,\frac{\ln(s+1)}{\ln 10}}{s+1}\, ds = \frac{2}{\ln 10}\int_0^9 \frac{\ln(s+1)}{s+1}\, ds$$

$$= \frac{\cancel{2}}{\ln 10}\left[\frac{[\ln(s+1)]^2}{\cancel{2}}\right]_0^9 = \ln 10 .$$

2. Evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cosh(\tan\theta)\sec^2\theta\, d\theta = I$

Q55 page 442

Let $\boxed{u = \tan\theta}$, $\boxed{du = \sec^2\theta\, d\theta}$,

$$I = \int_{-1}^{1} \cosh u\, du = \sinh u \Big]_{-1}^{1} = \sinh(1) - \sinh(-1)$$

$$= \frac{e^1 - e^{-1}}{2} - \left(\frac{e^{-1} - e^{+1}}{2}\right) = e - e^{-1}.$$

3. Evaluate $\int \sec^3 x \tan^3 x\, dx$

Q 36 page 466

$$= \int \sec^2 x \tan^2 x \cdot \sec x \tan x\, dx$$

$$= \int \sec^2 x\,(\sec^2 x - 1)\sec x \tan x\, dx,$$

Let $\boxed{\begin{array}{l} u = \sec x \\ du = \sec x \tan x\, dx \end{array}}$

$$= \int u^2(u^2-1)\, du = \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C.$$

4. Find $\int \sin(\ln x^2)\,dx$ $= I$

$$\boxed{\begin{array}{ll} u = \sin(\ln x^2) & dv = dx \\ du = \cos(\ln x^2)\cdot\frac{1}{x^2}\,2x\,dx & v = x \end{array}}$$

then, $I = x\sin(\ln x^2) - 2\int \cos(\ln x^2)\,dx$

$$\boxed{\begin{array}{ll} u = \cos(\ln x^2) & dv = dx \\ du = -\sin(\ln x^2)\,\frac{1}{x^2}\cdot 2x\,dx & v = x \end{array}}$$

$$I = x\sin(\ln x^2) - 2\left[x\cos(\ln x^2) + 2\underbrace{\int \sin(\ln x^2)\,dx}_{I}\right]$$

$$I = x\sin(\ln x^2) - 2x\cos(\ln x^2) - 4I$$

$$\Rightarrow \quad 5I \quad I = \frac{1}{5}x\sin(\ln x^2) - \frac{2}{5}x\cos(\ln x^2).$$

5. Evaluate $\int_1^e \frac{dt}{t\sqrt{1+(\ln t)^2}} = I$

Q38 page 470

let $\boxed{u = \ln t, \quad du = \frac{1}{t}\,dt}$ $\quad u \geq 0$

$$I = \int_0^1 \frac{du}{\sqrt{1+u^2}}, \quad \text{now let } \boxed{\begin{array}{l} u = \tan\theta \\ du = \sec^2\theta\,d\theta \end{array}}, \quad \text{WHY } \boxed{0 \leq \theta \leq \frac{\pi}{4}}$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sec^2\theta\,d\theta}{\sec\theta}$$

$$= \int_0^{\frac{\pi}{4}} \sec\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta|\Big]_0^{\frac{\pi}{4}}$$

$$= \ln(1+\sqrt{2}).$$

Math 102

Quiz#3

Key "Z"

____
40

Name:

ID No.:

Serial No.:

1. Evaluate $I = \int_1^4 \frac{\ln 2 \log_2 s}{s} ds$.

Q 39 Page 426

$$I = \int_1^4 \frac{\cancel{\ln 2} \frac{\ln s}{\cancel{\ln 2}}}{s} ds = \int_1^4 \frac{\ln s}{s} ds = \frac{(\ln s)^2}{2} \Big]_1^4$$

$$= \frac{1}{2}\left[(\ln 4)^2 - (\ln 1)^2\right] = \frac{1}{2}(2\ln 2)^2 = 2\ln 2.$$

2. Evaluate $\int_0^{\pi/2} 2\sinh(\sin\theta)\cos\theta \, d\theta = I$

Q 56 page 442

let $\boxed{u = \sin\theta}$, $\boxed{du = \cos\theta \, d\theta}$,

$$I = \int_0^1 2\sinh u \, du = 2\cosh u \Big]_0^1$$

$$= 2(\cosh 1 - \cosh 0) = 2\left(\frac{e^1 + e^{-1}}{2} - 1\right)$$

$$= e^1 + e^{-1} - 2.$$

3. Evaluate $\int \sec^4 x \tan^2 x \, dx = I$

Q 38 page 466

$$I = \int \sec^2 x \tan^2 x \sec^2 x \, dx$$

$$= \int (1 + \tan^2 x) \tan^2 x \sec^2 dx$$, let $\boxed{\begin{array}{l} u = \tan x \\ du = \sec^2 x \, dx \end{array}}$

$$= \int (1 + u^2) u^2 \, du$$

$$= \frac{u^3}{3} + \frac{u^5}{5} + C = \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C.$$

4. Find $\int \sin\,(\ln x^2)\,dx$

See Key "1"

5. Evaluate $\int_{\ln\frac{3}{4}}^{\ln\frac{4}{3}} \frac{e^t dt}{(1+e^{2t})^{3/2}} = I$

$\boxed{u = e^t, \qquad du = e^t\,dt}$

Q 36 page 470

$t = \ln\frac{4}{3} \Rightarrow u = \frac{4}{3}$

$t = \ln\frac{3}{4} \Rightarrow u = \frac{3}{4}$

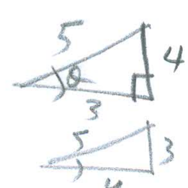
$$I = \int_{\frac{3}{4}}^{\frac{4}{3}} \frac{du}{(1+u^2)^{\frac{3}{2}}},$$

let $u = \tan\theta$,

$\boxed{\tan^{-1}\frac{3}{4} < \theta < \tan^{-1}\frac{4}{3}}$

$du = \sec^2\theta\, d\theta$

$$= \int_{\tan^{-1}\frac{3}{4}}^{\tan^{-1}\frac{4}{3}} \frac{\sec^2\theta}{\sec^3\theta}\, d\theta$$



$$= \int_{\tan^{-1}\frac{3}{4}}^{\tan^{-1}\frac{4}{3}} \cos\theta\, d\theta = \sin\theta\Big]_{\tan^{-1}\frac{3}{4}}^{\tan^{-1}\frac{4}{3}} = \sin\left(\tan^{-1}\frac{4}{3}\right) - \sin\left(\tan^{-1}\frac{3}{4}\right)$$

$$= \frac{4}{5} - \frac{3}{5} = \frac{1}{5}.$$