Math 102

Quiz#1

Key

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Name:

ID No.:

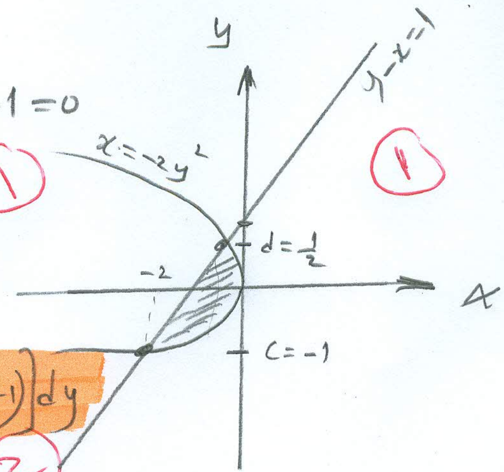
Serial No.:

1. Find the definite integral that represent the area enclosed by $y - x = 1$ and $x = -2y^2$. (Just set up the integral formula)

• Intersection pt(s): $\begin{cases} x = -2y^2 \\ x = y-1 \end{cases} \Rightarrow 2y^2 + y - 1 = 0$

$\Rightarrow \boxed{y = \frac{1}{2}}$ OR $\boxed{y = -1}$ 1

$\boxed{x = -\frac{1}{2}}$ OR $\boxed{x = -2}$

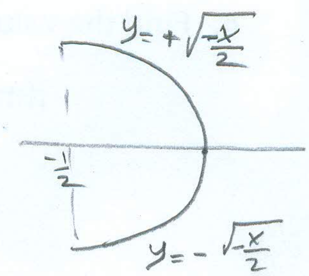


• w.r.t. y

$A = \int_c^d (x_{right} - x_{left})\,dy = \int_{-1}^{\frac{1}{2}} \left[(-2y)^2 - (y-1)\right] dy$ 2

OR

w.r.t. x



$$A = \int_{-2}^{-\frac{1}{2}} \left[(1+x) - \left(-\sqrt{\tfrac{-x}{2}}\right)\right] dx + \int_{-\frac{1}{2}}^{0} \left[\left(\sqrt{\tfrac{-x}{2}}\right) - \left(-\sqrt{\tfrac{-x}{2}}\right)\right] dx .$$

2. If $F(x) = \int_1^x f(z)dz$, where $f(x) = \int_1^{x^2} \frac{\sqrt{1+u^2}}{u} du$, find $\acute{F}(1)$.

• $F'(x) = \frac{d}{dx}\left(\int_1^x f(z)\,dz\right) = f(x)$ 1 $\boxed{\text{by FTC part "1"}}$

• $F''(x) = \frac{d}{dx}\left(f(x)\right) = \frac{d}{dx}\left(\int_1^{x^2} \frac{\sqrt{1+u^2}}{u} du\right) = \frac{\sqrt{1+(x^2)^2}}{x^2} \cdot (x^2)'$ 1

$= \frac{\sqrt{1+x^4}}{x^2} \cdot 2x = \frac{2\sqrt{1+x^4}}{x}$

$\Rightarrow \boxed{F''(1) = 2\sqrt{2}}$. 2

3. Evaluate $I = \int_0^{\frac{3\sqrt{2}}{4}} \frac{1}{\sqrt{9-4s^2}}\,ds$.

$$I = \int_0^{\frac{3\sqrt{2}}{4}} \frac{1}{\sqrt{9\left(1-\left(\frac{2s}{3}\right)^2\right)}}\,ds = \frac{1}{3}\int_0^{\frac{3\sqrt{2}}{4}} \frac{1}{\sqrt{1-\left(\frac{2s}{3}\right)^2}}\,ds \quad 1$$

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let $\boxed{u = \frac{2s}{3}}$

$\boxed{du = \frac{2}{3}\,ds}$

if $\boxed{s=0}$, $\boxed{u=0}$

if $\boxed{s=\frac{3\sqrt{2}}{4}}$, $\boxed{u=\frac{\sqrt{2}}{2}}$

$$= \frac{1}{\cancel{3}}\int_0^{\frac{\sqrt{2}}{2}} \frac{\frac{\cancel{3}}{2}}{\sqrt{1-u^2}}\,du = \frac{1}{2}\sin^{-1}u\Big]_0^{\frac{\sqrt{2}}{2}} \quad 1$$

$$= \frac{1}{2}\left[\sin^{-1}\frac{\sqrt{2}}{2} - \sin^{-1}0\right] = \frac{1}{2}\cdot\frac{\pi}{4} = \boxed{\frac{\pi}{8}}. \quad 1$$

4. Find the value of the following limit:

$$\lim_{n\to\infty}\left\{\sum_{i=1}^{n}\left[\left(\frac{\pi}{4n}\right)\left(\cos\frac{i\pi}{2n}\right)^2\right]\right\} \text{ on } \left[0,\frac{\pi}{2}\right].$$

$$\Delta x = \frac{b-a}{n} = \frac{\frac{\pi}{2}-0}{n} = \frac{\pi}{2n} \quad 1$$

1 $f(c_i) = \left(\cos\frac{i\pi}{2n}\right)^2$ and $\boxed{c_i = \frac{i\pi}{2n}}$ 1 "Rightend point"

Therefore, $f(x) = (\cos x)^2$

$$\lim_{n\to\infty}\left(\sum\right) = \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos^2 x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{2}}\left(\frac{1+\cos 2x}{2}\right)dx \quad 1$$

$$= \frac{1}{2}\cdot\frac{1}{2}\left\{\left[x + \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}\right\} = \frac{1}{4}\left(\frac{\pi}{2}\right) = \boxed{\frac{\pi}{8}} \quad 1$$

5. If f is a continuous function on $[0,1]$ and $\int_0^1 f(x)dx = 2$, find $\int_0^1 f(1-x)dx$.

Think by Substitution !!

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For $\int_0^1 f(1-x)\,dx$, let $\boxed{1-x=u}$ then $\boxed{-dx = du}$

and $\boxed{\begin{array}{l} x=0,\ u=1 \\ x=1,\ u=0 \end{array}}$

$$\text{So } \int_0^1 f(1-x)\,dx = -\int_1^0 f(u)\,du \quad 1$$

$$= \int_0^1 f(u)\,du = 2. \quad 2$$