March 18,2013

QUIZ♯3 Math102, sec 7

Net Time Allowed: 25 minutes

Name: **ID ♯:** **Serial:**

Exercise1:(06 points)

Evaluate

$$I=\int_0^{\sqrt[4]{\frac{\pi}{4}}} t^3\sec^4(t^4)\tan^4(t^4)\,dt$$

Let $x=t^4$ (01), then $dx=4t^3dt$: $\begin{vmatrix} t=0 \Rightarrow x=0 \\ t=\sqrt[4]{\frac{\pi}{4}} \Rightarrow x=\frac{\pi}{4}\end{vmatrix}$ (0.5), Thus

$$I=\frac{1}{4}\int_0^{\frac{\pi}{4}}\sec^4x\tan^4x\,dx \quad (01)$$

$$=\frac{1}{4}\int_0^{\frac{\pi}{4}}\sec^2x\tan^4x\sec^2x\,dx$$

$$=\frac{1}{4}\int_0^{4}(1+\tan^2x)\tan^4x\sec^2x\,dx \quad (01)$$

. Let $\begin{vmatrix} u=\tan x \Rightarrow du=\sec^2x\,dx \\ x=0 \Rightarrow u=0 \\ x=\frac{\pi}{4} \Rightarrow u=1\end{vmatrix}$ (01), Hence

$$I=\frac{1}{4}\int_0^1(1+u^2)u^4\,du \quad (01)=\frac{1}{4}\int_0^1(u^4+u^6)\,du \quad (0.5)=\frac{3}{35}$$

Exercise2:(04 points)

Evaluate

$$I=\int(x\ln x)^2\,dx=\int x^2(\ln x)^2\,dx$$

Let $\begin{vmatrix} u=(\ln x)^2 \\ dv=x^2dx\end{vmatrix} \Rightarrow \begin{vmatrix} du=2\frac{\ln x}{x}dx \\ v=\frac{1}{3}x^3\end{vmatrix}$ (01), Therefore

$$I=\frac{1}{3}x^3(\ln x)^2-\frac{2}{3}\int x^2\ln x\,dx \quad (01)$$

, again let $\begin{vmatrix} u=\ln x \to du=\frac{dx}{x} \\ dv=x^2dx \to v=\frac{x^3}{3}\end{vmatrix}$

$$I=\frac{1}{3}x^3(\ln x)^2-\frac{2}{3}\left[\frac{1}{3}x^3\ln x-\frac{1}{3}\int x^2dx\right] \quad (01)$$

$$=\frac{1}{3}x^3(\ln x)^2-\frac{2}{3}\left[\frac{1}{3}x^3\ln x-\frac{1}{9}x^3\right]+C$$

, Hence

$$I=\frac{1}{3}x^3(\ln x)^2-\frac{2}{9}x^3\ln x+\frac{2}{27}x^3+C \quad (01)$$