February 20,2013

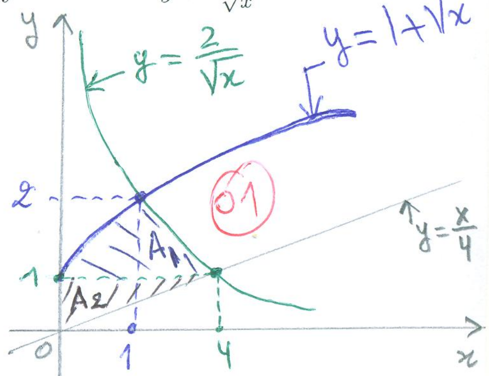
QUIZ♯2 Math102, sec 7

Net Time Allowed: 25 minutes

Name: **ID ♯:** **Serial:**

Exercise1: (07pts)

Find the area of the region in the 1^{st} quadrant bounded on the left by the y-axis, below by the line $y = \frac{x}{4}$, above left by the curve $y = 1+\sqrt{x}$, and above right by the curve $y = \frac{2}{\sqrt{x}}$.



• Intersection points: $\frac{2}{\sqrt{x}} = \frac{x}{4} \Rightarrow x^{\frac{3}{2}} = 8$ i.e $x = 4$. 0.5

⊛ A_2 is a triangle with area: $A_2 = (1\times4)\frac{1}{2} = 2$. 01

⊛ To find A_1 we need to integrate w.r.to y:

on $\frac{2}{\sqrt{x}} = 1+\sqrt{x} \Rightarrow x = 1$ thus the intersection point is $(1,2)$. 0.5

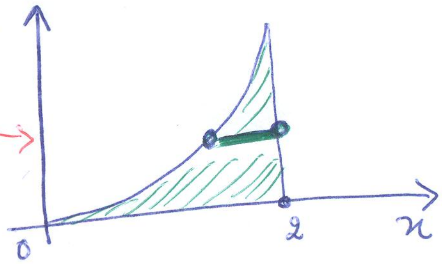
Now clearly $y = \frac{2}{\sqrt{x}}$ is on the right side of $y = 1+\sqrt{x}$.

The Area: $A_1 = \int_1^2 \left[\frac{4}{y^2} - (y-1)^2\right] dy = \left[-\frac{4}{y} - \frac{(y-1)^3}{3}\right]_1^2 = \frac{5}{3}$. 02 01

Hence the Total area is: $A = A_1 + A_2 = \frac{5}{3} + 2 = \frac{11}{3}$ 01

Exercise2: (03pts)

Find the volume of the solid generated by revolving the region in the first quadrant bounded above by the parabola $y = x^2$, below by the x-axis and on the right by the line $x = 2$ about the y-axis.



$$V = \int_0^4 \pi \left[(R(y))^2 - (r(y))^2\right] dy$$ 01

$$= \pi \int_0^4 (4-y)\, dy$$ 01

$$= \pi \left[4y - \frac{y^2}{2}\right]_0^4 = \pi(16-8) = 8\pi.$$ 01