

## EXERCISE Math102/

Name:

ID #:

section:

The German mathematician Karl Weierstrass (1815-1897) noticed that the substitution  $t = \tan\left(\frac{x}{2}\right)$  will convert any rational function of  $\sin x$  and  $\cos x$  into an ordinary rational function of  $t$ .

a) If  $t = \tan\left(\frac{x}{2}\right)$ ,  $-\pi < x < \pi$ , sketch a right triangle or use trigonometric identities to show that

$$\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1+t^2}}, \quad \sin\left(\frac{x}{2}\right) = \frac{t}{\sqrt{1+t^2}}$$

b) Show that

$$\cos x = \frac{1-t^2}{1+t^2}, \quad \sin x = \frac{2t}{1+t^2}$$

c) Show that

$$dx = \frac{2}{1+t^2} dt$$

Application:

Evaluate:

$$I = \int \frac{dx}{5+3\cos x}$$