Exercise: Show whether or not:

$$\int_{1}^{+\infty} \frac{dx}{(e^{x}-x)^{\frac{1}{2}}}$$ is Convergent or divergent.

Solution: we have $e^{x}-x > x^{p}$ for $p \geq 3$ and x Large enough.

Thus $\frac{1}{(e^{x}-x)^{1/2}} < \frac{1}{x^{p/2}}$, Therefore:

$$\int_{1}^{+\infty} \frac{dx}{(e^{x}-x)^{1/2}} < \int_{1}^{+\infty} \frac{dx}{x^{p/2}}$$

Since $\int_{1}^{+\infty} \frac{dx}{x^{p/2}}$ is Convergent (Use Integral Test! HW)
$(p \geq 3)$

Hence $\int_{1}^{+\infty} \frac{dx}{(e^{x}-x)^{1/2}}$ is Convergent.