MATH 102.5 (Term 122)

Quiz 6 (Sects. 10.4-10.6) Duration: 20mn

Name: ID number:

1.)(2pts)Use comparison test to show that the series $\sum_{n=1}^{\infty} \frac{\cos^4(n^2+1)}{(n+1)^{3/2}}$ converges.

2.)(2pts)Use ratio test to study the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$.

3.)(4pts)Do the series $\sum_{n=1}^{\infty} \left(\frac{4+5\ln\sqrt{n}}{\sqrt{n}+7}\right)^n$ and $\sum_{n=1}^{\infty} \left(\frac{6+3n^2}{2n^2+n}\right)^n$ converge or diverge?

4.)(2pts)Find the smallest number of terms required to approximate the sum $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^4-1}$ so that $|\text{error}| < 0.0001$.

1.) $\frac{\cos^4(n^2+1)}{(n+1)^{3/2}} \leq \frac{1}{(n+1)^{3/2}}$

and the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)^{3/2}}$ Converges

$\Longrightarrow \sum_{n=1}^{\infty} \frac{\cos^4(n^2+1)}{(n+1)^{3/2}}$ also Converges

2) $\lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{n+1}} \frac{n^n}{n!} = \lim_{n\to\infty} \frac{n+1 \; n^n}{(n+1)^{n+1}}$

$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} e^{n\ln\left(\frac{n}{n+1}\right)}$

$= e^{-1},$

Because

$\lim_{n\to\infty} n\ln\left(\frac{n}{n+1}\right) = \lim_{n\to\infty} \frac{\ln\left(\frac{n}{n+1}\right)}{\frac{1}{n}}$

$= \lim_{n\to\infty} \frac{\frac{1}{n(n+1)}}{-\frac{1}{n^2}}$ (Hopital rule)

$= -1$

$\boxed{\text{or}}$ $\lim_{n\to\infty} n\ln\left(\frac{n+1}{n}\right) = \lim_{n\to\infty} \frac{\ln(1+\frac{1}{n})}{\frac{1}{n}}$

$= \lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$

Since $\frac{1}{e} < 1$, the series $\sum \frac{n!}{n^n}$ converges

3.) $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{4+5\ln\sqrt{n}}{\sqrt{n}+7} = 0 < 1$

$\Rightarrow \sum \left(\frac{4+5\ln\sqrt{n}}{\sqrt{n}+7}\right)^n$ Converges

$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{6+3n^2}{2n^2+n} = 3/2 > 1$

$\Rightarrow \sum \left(\frac{6+3n^2}{2n^2+n}\right)^n$ diverges

4.) $\frac{1}{(n+1)^4-1} < 0.0001 = 10^{-4}$

$(n+1)^4 - 1 > 10^4$

$(n+1)^4 > 10^4 + 1 > 10^4$

$n+1 > 10$

$\Rightarrow n > 9$

$n = 10$ is the smallest number satisfying the condition