MATH 102.5 (Term 122)

Quiz 5 (Sects. 10.1-10.3) Duration: 20mn

Name: ID number:

1.)(3pts)Find the limit of the sequence $\sqrt{2}(\sqrt{12}-\sqrt{2})$, $\sqrt{3}(\sqrt{13}-\sqrt{3})$, $\sqrt{4}(\sqrt{14}-\sqrt{4})$, $\sqrt{5}(\sqrt{15}-\sqrt{5})$,...

2.)(3pts)What is the value of sum $\sum_{n=1}^{\infty} \frac{e^{1-2n}}{(\sqrt{2})^{2-2n}}$.

3.)(4pts)Do the the following series converge or diverge $\sum_{n=1}^{\infty} \frac{4n}{(n^2+1)^2}$, $\sum_{n=2}^{\infty} \frac{2}{n(\ln n)^2}$ (Hint: use the integral test for both series).

1) $a_n = \sqrt{n}\left(\sqrt{n+10}-\sqrt{n}\right)$, $n=2,3,\dots$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt{n}\left(\sqrt{n+10}-\sqrt{n}\right)$$

$$= \lim_{n\to\infty} \frac{\sqrt{n}\left(\sqrt{n+10}-\sqrt{n}\right)\left(\sqrt{n+10}+\sqrt{n}\right)}{\sqrt{n+10}+\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n}\left((n+10)-n\right)}{\sqrt{n+10}+\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{10\sqrt{n}}{\sqrt{n+10}+\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{\cancel{\sqrt{n}}\, 10}{\cancel{\sqrt{n}}\left(\sqrt{1+\frac{10}{n}}+1\right)}$$

$$= 5$$

2)
$$\sum_{n=1}^{\infty} \frac{e^{1-2n}}{(\sqrt{2})^{2-2n}} = \sum_{n=1}^{\infty} \frac{e^{1-2n}}{2^{1-n}}$$

$$= \sum_{n=1}^{\infty} \frac{e}{2}\left(\frac{e^{-2}}{2^{-1}}\right)^n = \sum_{n=1}^{\infty} \frac{e}{2}\left(\frac{2}{e^2}\right)^n$$

$$= \sum_{n=1}^{\infty} \frac{e}{2}\,\frac{2}{e^2}\left(\frac{2}{e^2}\right)^{n-1}$$

$$= \sum_{n=1}^{\infty} \frac{1}{e}\left(\frac{2}{e^2}\right)^{n-1} = \frac{\frac{1}{e}}{1-\frac{2}{e^2}} = \frac{e}{e^2-2}$$

3)
$$\int_1^{\infty} \frac{4x}{(x^2+1)^2}\,dx = \lim_{b\to\infty}\left[-\frac{2}{x^2+1}\right]_1^b$$

$$= \lim_{b\to\infty}\left(\frac{-2}{b^2+1}+1\right)$$

$$= 1$$

Thus, the series $\sum_{n=1}^{\infty} \frac{4n}{(n^2+1)^2}$ Converges

$$\int_2^{\infty} \frac{2}{x(\ln x)^2}\,dx = \int_{\ln 2}^{\infty} \frac{2\,du}{u^2}$$

$$= \lim_{b\to\infty}\left[-\frac{2}{u}\right]_{\ln 2}^{b}$$

$$= \frac{2}{\ln 2}$$

The series $\sum_{n=2}^{\infty} \frac{2}{n(\ln n)^2}$ Converges