Quiz 4 (122) Math 102

1) Evaluate $I=\int_1^e x^3(\ln x)^2\,dx$, or, $I=\int_0^1 x(x+1)e^x\,dx$

2) Evaluate $J=\int(\sec\theta+\sin\theta)^2\,d\theta$

3) Evalute $K=\int\frac{\sin^{-1}x}{x^2}\,dx$

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1) $I=\int_1^e x^3(\ln x)^2\,dx$

$u=(\ln x)^2 \to u'=\frac{2\ln x}{x}$

$v'=x^3 \to v=\frac{x^4}{4}$

$I=\left[\frac{x^4}{4}(\ln x)^2\right]_1^e-\frac{1}{2}\int_1^e x^3\ln x\,dx$

$u=\ln x \to u'=\frac{1}{x}$

$v'=x^3 \to v=\frac{x^4}{4}$

$I=\left[\frac{x^4}{4}(\ln x)^2-\frac{x^4}{8}\ln x\right]_1^e+\frac{1}{8}\int_1^e x^3\,dx$

$=\left[\frac{x^4}{4}(\ln x)^2-\frac{x^4}{8}\ln x+\frac{1}{32}x^4\right]_1^e$

$=\left(\frac{e^4}{4}-\frac{e^4}{8}+\frac{e^4}{32}\right)-\frac{1}{32}$

$=\frac{5e^4-1}{32}$

Or

$I=\int_0^1 x(x+1)e^x\,dx$

$u=x(x+1) \to u'=2x+1$

$v'=e^x \to v=e^x$

$I=\left[x(x+1)e^x\right]_0^1-\int_0^1(2x+1)e^x\,dx$

$u=2x+1 \to u'=2$

$v'=e^x \to v=e^x$

$I=\left[x(x+1)e^x-(2x+1)e^x\right]_0^1+\int_0^1 2e^x\,dx$

$=\left[\left(x(x+1)-(2x+1)+2\right)e^x\right]_0^1$

$=\left[\left(x^2-x+1\right)e^x\right]_0^1$

$=e-1$

2) $J=\int(\sec\theta+\sin\theta)^2\,d\theta$

$=\int(\sec^2\theta+2\tan\theta+\sin^2\theta)\,d\theta$

$=\tan\theta+2\ln|\sec\theta|+\int\frac{1-\cos 2\theta}{2}\,d\theta$

$=\tan\theta+2\ln|\sec\theta|+\frac{\theta}{2}-\frac{\sin 2\theta}{4}+C$

3) $K=\int\frac{\sin^{-1}x}{x^2}\,dx$

$u=\sin^{-1}x \to u'=\frac{1}{\sqrt{1-x^2}}$

$v'=\frac{1}{x^2} \to v=\frac{-1}{x}$

$$K = -\frac{\sin^{-1}x}{x} + \int \frac{1}{x\sqrt{1-x^2}}\,dx$$

let $x = \sin\theta$, $-\pi/2 < \theta < \pi/2$

$dx = \cos\theta\, d\theta$

Thus,

$$K = -\frac{\sin^{-1}x}{x} + \int \frac{\cos\theta\, d\theta}{\sin\theta\sqrt{\cos\theta}}$$

$$= -\frac{\sin^{-1}x}{x} + \int \frac{\cos\theta}{\sin\theta\cos\theta}\,d\theta$$

$$= -\frac{\sin^{-1}x}{x} + \int \frac{d\theta}{\sin\theta}$$

$$= -\frac{\sin^{-1}x}{x} - \ln\left|\underbrace{\csc\theta}_{\frac{1}{x}} + \underbrace{\cot\theta}_{= \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{1-\sin^2\theta}}{\sin\theta} = \frac{\sqrt{1-x^2}}{x}}\right| + C$$

So, $$K = -\frac{\sin^{-1}x}{x} - \ln\left|\frac{1}{x} + \frac{\sqrt{1-x^2}}{x}\right| + C$$

OR

$$= -\frac{\sin^{-1}x}{x} - \operatorname{sech}^{-1}x + C$$