MATH 102.5 (Term 122)

Quiz 3 (Sects. 6.4, 7.1, 7.3) Duration: 20mn

Name: ID number:

1.)(4pts)Find the area of the surface generated by rotating the curve $y = 2\sqrt{x}$, $8 \le x \le 15$, about the x-axis.

2.)(3pts)Solve the equation $2\sinh x = 1$, $x \in \mathbb{R}$.

3.)(3pts)Evaluate the integral $\int \frac{e^{2x}}{\sqrt{e^{4x}+4e^{2x}-12}}dx$, $x > \frac{\ln 2}{2}$.

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1) Let $y = f(x) = 2\sqrt{x}$

$$A = 2\pi \int_8^{15} f(x)\sqrt{1+f'(x)^2}\,dx$$

$$f(x) = \frac{1}{\sqrt{x}}$$

$$A = 2\pi \int_8^{15} 2\sqrt{x}\sqrt{1+\frac{1}{x}}\,dx$$

$$= 4\pi \int_8^{15} \sqrt{x+1}\,dx$$

$$= 4\pi \cdot \frac{2}{3}\left[(x+1)^{3/2}\right]_8^{15}$$

$$= \frac{8\pi}{3}\left(16^{3/2} - 9^{3/2}\right)$$

$$= \frac{8\pi}{3}\left(4^3 - 3^3\right)$$

$$= \frac{8\pi}{3} \cdot 37 = \frac{296\pi}{3}$$

2) $2\sinh x = 1$

$$2\left(\frac{e^x - e^{-x}}{2}\right) = 1$$

$$e^x - e^{-x} = 1$$

$$e^{2x} - e^x - 1 = 0$$

Let $u = e^x \Rightarrow u^2 - u - 1 = 0$

$$\Delta = 1 + 4 = 5$$

$$u_1 = \frac{1-\sqrt{5}}{2}, \quad u_2 = \frac{1+\sqrt{5}}{2}$$

$\Rightarrow e^x = \frac{1-\sqrt{5}}{2} < 0$ (rejected)

$e^x = \frac{1+\sqrt{5}}{2} \Rightarrow \boxed{x = \ln\left(\frac{1+\sqrt{5}}{2}\right)}$

3) $$I = \int \frac{e^{2x}}{\sqrt{(e^{2x}+2)^2 - 16}}\,dx$$

Let $u = e^{2x}+2$, $du = 2e^{2x}dx$

$$I = \frac{1}{2}\int \frac{du}{\sqrt{u^2-16}}$$

$$= \frac{1}{2}\cosh^{-1}\left(\frac{u}{4}\right) + C$$

$$= \frac{1}{2}\cosh^{-1}\left(\frac{e^{2x}+2}{4}\right) + C$$