

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 101
Final Exam
Term 122
Monday 20/15/2013
Net Time Allowed: 180 minutes

MASTER VERSION

1. An equation of the tangent line to the curve $y = \frac{1}{x^3}$ when $x = -1$ is given by

(a) $y = -3x - 4$

(b) $y = -2x - 3$

(c) $y = -\frac{1}{3}x - \frac{4}{3}$

(d) $y = \frac{1}{3}x + 1$

(e) $y = -3x - 2$

2. Let μ be a real number not equal to -1 . Then $\lim_{x \rightarrow 1} \frac{x^{\frac{\mu}{\mu+1}} - 1}{x - 1} =$

(a) $\frac{\mu}{\mu + 1}$

(b) $\frac{\mu + 1}{\mu}$

(c) $\frac{1}{\mu + 1}$

(d) $\mu + 1$

(e) does not exist

3. $\sum_{k=1}^7 (2k + 6k^2) =$

(a) (16)(7)(8)

(b) (15)(7)(8)

(c) (16)(7)

(d) (7)(8)(10)

(e) (7)(8)(14)

4. Using four rectangles and the **midpoint rule**, the area under the graph of $f(x) = 1 + x^2$ from $x = 0$ to $x = 4$ is approximately equal to

(a) 25

(b) 20

(c) 30

(d) 27

(e) 18

5. Suppose that $g(x) \leq f(x) \leq h(x)$ for all $x \neq 2$ and suppose that

$$\lim_{x \rightarrow 2} g(x) = \lim_{x \rightarrow 2} h(x) = -5.$$

Which one of the following statements is **TRUE**:

- (a) $\lim_{x \rightarrow 2} f(x) \neq 0$
 - (b) $f(2)$ must be equal to -5
 - (c) $f(2)$ cannot be equal to 0
 - (d) $\lim_{x \rightarrow 2} f(x)$ can be different from -5
 - (e) $f(2)$ and $\lim_{x \rightarrow 2} f(x)$ must be equal
6. If $f(x) = \frac{x^3 + 2x^2 - 1}{(x + 1)^2}$, then an equation of the oblique asymptote for the graph of f is

- (a) $y - x = 0$
- (b) $y - x - 1 = 0$
- (c) $y + x = 0$
- (d) $y + x + 1 = 0$
- (e) f does not have an oblique asymptote

7. If $f(x) = \cot^{-1}\left(\frac{1}{x}\right) - \tan^{-1}x$, then $f'(1) =$

(a) $f'(2)$

(b) does not exist

(c) -1

(d) $\frac{-2}{3}$

(e) $f'(0)$

8. If $y = (\sec x + \tan x)(\sec x - \tan x)$, then $\frac{dy}{dx} =$

(a) 0

(b) $\sec^2 x$

(c) $\sec^3 x$

(d) 1

(e) $\sec^2 x \tan x$

9. The slope of the **normal line** to the curve $2y + \pi \sin(xy) = 2\pi$ at the point $(1, \pi)$ is

(a) $\frac{\pi - 2}{\pi^2}$

(b) 0

(c) $\frac{1}{\pi}$

(d) $-\frac{2}{\pi^2}$

(e) None of them

10. Newton's method is used to estimate the x -coordinate of the point of intersection of the curves $y = \sqrt{x}$ and $y = 1 - x^2$. If we start with $x_0 = 1$, then $x_1 =$

(a) $\frac{3}{5}$

(b) 0

(c) $\frac{-1}{2}$

(d) $\frac{1}{2}$

(e) $\frac{8}{5}$

11. If the function

$$f(x) = \begin{cases} \frac{x+b}{b+1} & x < 0 \\ x^2 + b & x \geq 0 \end{cases}$$

is continuous everywhere, then $f(-1) =$

- (a) -1
- (b) 0
- (c) 2
- (d) 4
- (e) -3

12. Let $f(x) = \sqrt{1 - 3x}$. The **greatest possible** value of $\delta > 0$ for which $\lim_{x \rightarrow -1} f(x) = 2$, when $\varepsilon = \frac{1}{2}$ is

- (a) $\frac{7}{12}$
- (b) $\frac{9}{12}$
- (c) $\frac{5}{12}$
- (d) $\frac{5}{2}$
- (e) $\frac{3}{2}$

13. Evaluate the limit $\lim_{x \rightarrow -\infty} (x + \sqrt{x^2 + 2x})$

(a) -1

(b) 1

(c) -2

(d) 2

(e) 0

14. If $y = (2x)^{\sin(2x)}$, $\frac{dy}{dx} =$

(a) $y \left[\frac{\sin(2x)}{x} + 2 \ln(2x) \cos(2x) \right]$

(b) $y \left[\frac{\sin(2x)}{2x} + 2 \ln(2x) \cos(2x) \right]$

(c) $y \left[\frac{\sin(2x)}{x} - 2 \ln(2x) \cos(2x) \right]$

(d) $\sin(2x) (2x)^{\sin(2x)-1} \ln(2x)$

(e) $y \left[\frac{\cos(2x)}{x} - 2 \ln(2x) \sin(2x) \right]$

15. Evaluate the limit $\lim_{x \rightarrow -1^+} (\sqrt{x+1} \ln(x+1))$
- (a) 0
 - (b) 1
 - (c) -1
 - (d) ∞
 - (e) $-\infty$
16. Let $f(x) = \cos^2 x + \sin x$, $0 < x < \pi$. Which one of the following statements is **TRUE**:
- (a) f is decreasing on the intervals $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$ and $\left(\frac{5\pi}{6}, \pi\right)$
 - (b) f is decreasing on the interval $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$
 - (c) f is increasing on the interval $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$
 - (d) f is increasing on the intervals $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
 - (e) f is decreasing on the intervals $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$ and $\left(\frac{2\pi}{3}, \pi\right)$

17. If $F(x) = f(xf(xf(x)))$ where $f(1) = 2$, $f(2) = 3$, $f'(1) = 4$, $f'(2) = 5$ and $f'(3) = 6$, then $F'(1) =$

(a) 198

(b) 0

(c) -1

(d) -200

(e) 144

18. Let $f(x) = \frac{2x^2}{x^2 - 1}$. Which one of the following statements is **TRUE**:

(a) The graph of f is concave up on $(-\infty, -1) \cup (1, \infty)$

(b) The graph of f is concave up on $(-\infty, -1) \cup (0, 1)$

(c) The graph of f is concave down on $(-1, 1) \cup (1, \infty)$

(d) The graph of f is concave down on $(-1, 0) \cup (1, \infty)$

(e) f has two inflection points

19. Let $f(x) = x^2 \sqrt{5 - \frac{x}{2}}$, $x \in [-2, 2]$. Which one of the following statements is **TRUE**?

- (a) f has a local minimum at $x = 0$
- (b) f has a local maximum at $x = 8$
- (c) the absolute maximum value of f is 8
- (d) f has absolute minimum at $x = -2$
- (e) f has no absolute minimum

20. The most general antiderivative of $f(t) = \frac{te^{2t} + \sqrt[3]{t}}{t}$ is

- (a) $\frac{1}{2}e^{2t} + 3t^{1/3} + C$
- (b) $e^{2t} + 3t^{2/3} + C$
- (c) $te^{2t} + t^{-2/3} + C$
- (d) $2e^{2t} - \frac{3}{2}t^{-5/3} + C$
- (e) $\frac{1}{2}e^{2t} - \frac{3}{2}t^{-5/3} + C$

21. The **number** of the critical points of $f(x) = |x^3 - 4x|$ is

(a) 5

(b) 4

(c) 3

(d) 2

(e) 1

22. The real values of x_0 and L that satisfy the following limit

$$\lim_{x \rightarrow x_0} \frac{\ln(x+1)}{x-x_0} = 3L$$

are

(a) $x_0 = 0; L = \frac{1}{3}$

(b) $x_0 = -1; L = \frac{1}{2}$

(c) $x_0 = -2; L = 1$

(d) $x_0 = 1; L = \frac{1}{3}$

(e) $x_0 = \frac{1}{3}; L = \frac{1}{4}$

23. Approximating $\tan^{-1}(1.01)$ using a linearization of $f(x) = \tan^{-1}(x)$ at a suitably chosen integer near 1.01 is equal to

(a) $\frac{\pi}{4} + 0.005$

(b) $\frac{\pi}{2} + 0.005$

(c) $\frac{\pi}{4} + 0.01$

(d) $\frac{\pi}{2} + 0.01$

(e) 0.005

24. The value(s) of c satisfying the conclusion of the Mean Value Theorem for the function $f(x) = x + \frac{1}{x}$, on the interval $\left[\frac{1}{2}, 2\right]$ is (are)

(a) 1

(b) -1 and 1

(c) $\frac{1}{2}$ and $\frac{3}{2}$

(d) $\frac{1}{4}$ and 1

(e) 1 and $\frac{7}{4}$

25. Let $a > 0$ and let $f(x) = \frac{x^2}{3} + \frac{x}{a}$, $a \leq x \leq 2a$. The value of a such that the **average rate of change** of the function f on the interval $[a, 2a]$ is the **smallest possible** is

(a) 1

(b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$

(d) 2

(e) $\sqrt{3}$

26. A surveyor, standing 50ft from the base of a building, measures the angle of elevation to the top of the building to be 45° . How accurately must the angle be measured for the percentage error in estimating the height of the building to be less than 3%?

(a) 1.5%

(b) 1%

(c) 2%

(d) 2.5%

(e) 3%

27. The volume of a cube is increasing at the rate of $270 \text{ cm}^3/\text{min}$ at the instant its edges are 6 cm long. At the same instant, the rate at which the lengths of the edges is changing is equal to
- (a) $2.5 \text{ cm}/\text{min}$
 - (b) $3 \text{ cm}/\text{min}$
 - (c) $3.5 \text{ cm}/\text{min}$
 - (d) $2 \text{ cm}/\text{min}$
 - (e) $4 \text{ cm}/\text{min}$
28. The least amount of material needed to construct an open-top right circular can that will hold a volume of 1000 cm^3 is equal to
- (a) $300\pi^{1/3} \text{ cm}^2$
 - (b) $100\pi^{-2/3} \text{ cm}^2$
 - (c) $10\pi^{-1/3} \text{ cm}^2$
 - (d) $400\pi^{1/3} \text{ cm}^2$
 - (e) $300\pi^{-2/3} \text{ cm}^2$

Q	MM	V1	V2	V3	V4
1	a	a	a	e	a
2	a	d	c	d	c
3	a	e	e	e	d
4	a	a	d	c	b
5	a	a	d	c	b
6	a	d	a	d	b
7	a	a	e	a	c
8	a	a	e	b	e
9	a	c	a	d	b
10	a	b	a	c	d
11	a	a	c	c	c
12	a	c	e	a	d
13	a	d	b	c	e
14	a	a	c	a	d
15	a	c	c	a	b
16	a	b	b	c	b
17	a	a	e	c	d
18	a	c	c	c	d
19	a	a	d	c	a
20	a	d	d	c	a
21	a	d	e	b	a
22	a	b	e	a	e
23	a	c	d	a	c
24	a	b	e	d	c
25	a	b	a	b	c
26	a	a	a	c	a
27	a	c	b	b	b
28	a	a	b	e	e