King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102 – Term 122 – Exam II

Duration: 100 minutes

Name: Key ID Number:

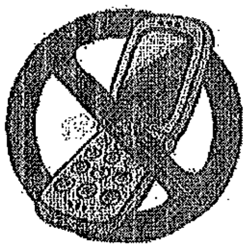
Section Number: Serial Number:

Class Time: Instructor's Name:

Instructions:

1. Calculators and Mobiles are not allowed.
2. Write neatly and eligibly. You may lose points for messy work.
3. Show all your work. No points for answers without justification.
4. Make sure that you have 6 pages of problems.

Page Number	Points	Maximum Points
1		20
2		16
3		17
4		16
5		14
6		17
Total		100



(Q1) (8-points) Write the form of the partial fraction decomposition of

$$\frac{3x}{(x-1)^2(x^2+1)(x^2+x+1)^2}.$$

DO NOT EVALUATE THE COEFFICIENTS

$$\frac{3X}{(x-1)^2(x^2+1)(x^2+x+1)^2} = \frac{A}{X-1} + \frac{B}{(X-1)^2} + \frac{CX+D}{x^2+1} + \frac{EX+F}{X^2+X+1} + \frac{GX+H}{(X^2+X+1)^2}$$

(1 pt) (1 pt) (2 pts) (2 pts) (2 pts)

(Q2) (6-points) If $\sinh x = \frac{-3}{4}$, then find the value of $\operatorname{sech} x$.

$$\cosh X = \sqrt{1+\sinh^2 X} \quad \text{(2 pts)}$$

$$= \sqrt{1+\frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4} \quad \text{(2 pts)}$$

$$\therefore \operatorname{Sech} X = \frac{1}{\cosh X} = \frac{4}{5} \quad \text{(2 pts)}$$

(Q3) (6-points) Find the area of the region bounded by the curve $y = x\sec^2 x$ and the lines $x = 0, x = \frac{\pi}{4}$, and $y = 0$.

$$\text{Area} = \int_0^{\frac{\pi}{4}} X \sec^2 X \, dX \quad \text{(1 pt)} \qquad \text{let } u = X, \quad dv = \sec^2 X \, dX \quad \text{(2 pts)}$$

$$du = dX, \quad V = \tan X$$

$$\Rightarrow \text{Area} = X \tan X \Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan X \, dX \quad \text{(2 pts)}$$

$$= X \tan X - \ln|\sec X| \Big|_0^{\frac{\pi}{4}} = \frac{\pi}{4} - \ln\sqrt{2} \quad \text{(1 pt)}$$

(Q4) (9-points) Find the area of the surface generated by revolving the curve $x = \sqrt{4y - y^2}, 1 \leq y \leq 2$ about the y-axis.

$$S = \int_1^2 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy \quad (2\text{ pts})$$

$$x = \sqrt{4y-y^2} \Rightarrow \frac{dx}{dy} = \frac{4-2y}{2\sqrt{4y-y^2}} = \frac{2-y}{\sqrt{4y-y^2}} \quad (2\text{ pts})$$

$$1+\left(\frac{dx}{dy}\right)^2 = 1 + \frac{4-4y+y^2}{4y-y^2} = \frac{4y-y^2+4-4y+y^2}{4y-y^2}$$

$$= \frac{4}{4y-y^2} \quad (2\text{ pts})$$

$$\therefore S = 2\pi \int_1^2 \sqrt{4y-y^2} \cdot \frac{2}{\sqrt{4y-y^2}}\, dy = 4\pi \int_1^2 dy = 4\pi \quad (3\text{ pts})$$

(Q5) (7-points) If $y = \ln(\cosh x) - \frac{1}{2}\tanh^2 x$, then find $\frac{dy}{dx}$ and write your answer in terms of $\tanh x$.

$$\frac{dy}{dx} = \frac{\sinh x}{\cosh x} - \tanh x \operatorname{sech}^2 x \quad (4\text{ pts})$$

$$= \tanh x - \tanh x \operatorname{sech}^2 x \quad (1\text{ pt})$$

$$= \tanh x \left(1 - \operatorname{sech}^2 x\right)$$

$$= \tanh^3 x. \quad (2\text{ pts})$$

(Q6) (5-points) Evaluate each of the following integrals

(A) $\displaystyle\int \frac{\cos x}{\sqrt{9+\sin^2 x}}\,dx$

let $u = \sin x \Rightarrow du = \cos x\, dx$ (2 pts)

$$\int \frac{\cos x}{\sqrt{9+\sin^2 x}}\,dx = \int \frac{1}{\sqrt{9+u^2}}\,du$$

$$= \sinh^{-1}\left(\frac{u}{3}\right) + C \quad (2\text{ pts})$$

$$= \sinh^{-1}\left(\frac{\sin x}{3}\right) + C \quad (1\text{ pt})$$

(B) (12-points) $\displaystyle\int e^{-x}\sin 2x\,dx$

$$I = \int e^{-x}\sin(2x)\,dx$$

let $u = \sin 2x$, $dv = e^{-x}dx$

$du = 2\cos(2x)dx$, $v = -e^{-x}$] (2 pts)

$$I = -e^{-x}\sin(2x) + 2\int e^{-x}\cos(2x)\,dx \quad (2\text{ pts})$$

let $u = \cos(2x)$, $dv = e^{-x}dx$

$du = -2\sin(2x)dx$, $v = -e^{-x}$] (2 pts)

(2 pts)

$$\Rightarrow I = -e^{-x}\sin(2x) - 2e^{-x}\cos(2x) - 4\int e^{-x}\sin(2x)\,dx$$

$$\Rightarrow 5I = -e^{-x}\sin(2x) - 2e^{-x}\cos(2x) \quad (2\text{ pts})$$

$$\Rightarrow I = -\frac{1}{5}e^{-x}\sin(2x) - \frac{2}{5}e^{-x}\cos(2x) + C \quad . \quad (2\text{ pts})$$

(C) (6-points) $\displaystyle\int \frac{dx}{\sqrt{x}\sqrt{1-x}}$ [Hint: You may use $u = \sqrt{1-x}$]

(2 pts)

$$u = \sqrt{1-x} \Rightarrow u^2 = 1-x \Rightarrow x = 1-u^2 \Rightarrow dx = -2u\,du$$

$$\int \frac{dx}{\sqrt{x}\,\sqrt{1-x}} = \int \frac{-2u\,du}{\sqrt{1-u^2}\;u} \quad \text{(2pts)}$$

$$= -2\int \frac{du}{\sqrt{1-u^2}} = -2\sin^{-1}u + C$$

$$= -2\sin^{-1}\sqrt{1-x} + C$$

(2 pts)

(D) (10-points) $\displaystyle\int \csc^6(2x)\,\cot^3(2x)\,dx$

$$I = \int \csc^6(2x)\cot^3(2x)\,dx$$

$$= \int \csc^5(2x)\cot^2(2x) \cdot \csc(2x)\cot(2x)\,dx \quad \text{(2 pts)}$$

$$= \int \csc^5(2x)\left[\csc^2(2x) - 1\right]\csc(2x)\cot(2x)\,dx \quad \text{(1 pt)}$$

let $u = \csc(2x) \Rightarrow du = -2\csc(2x)\cot(2x)\,dx$ (2 pts)

$$\Rightarrow I = -\frac{1}{2}\int u^5(u^2-1)\,du = -\frac{1}{2}\int (u^7 - u^5)\,du \quad \text{(2 pts)}$$

$$= -\frac{1}{16}u^8 + \frac{1}{12}u^6 + C = -\frac{1}{16}\csc^8(2x) + \frac{1}{12}\csc^6(2x) + C$$

(2 pts) (1 pt)

(E) (14-points) $\int \frac{x+2}{(x-1)(x^2+1)}\,dx$

$$\frac{x+2}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$\Rightarrow \quad x+2 = A(x^2+1) + (Bx+C)(x-1) \qquad (2\text{ pts})$$

$$x=1 : \quad 2A = 3 \Rightarrow A = \frac{3}{2}. \qquad (1\text{ pt})$$

$$\text{Coef } x^2 : \quad A+B=0 \Rightarrow B = -\tfrac{3}{2} \qquad (2\text{ pts})$$

$$\text{Constant term} : \quad A-C=2 \Rightarrow C = A-2 = \frac{3}{2}-2 = -\frac{1}{2}. \qquad (1\text{ pt})$$

$$\therefore \int \frac{x+2}{(x-1)(x^2+1)}\,dx$$

$$= \frac{3}{2}\int \frac{1}{x-1}\,dx - \int \frac{\frac{3}{2}x+\frac{1}{2}}{x^2+1}\,dx \qquad (2\text{ pts})$$

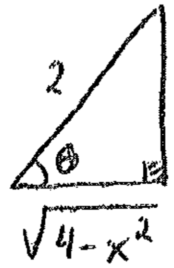
$$= \frac{3}{2}\ln|x-1| - \frac{3}{2}\int \frac{x}{x^2+1}\,dx - \frac{1}{2}\int \frac{1}{x^2+1}\,dx \qquad (2\text{ pts})$$

$$= \frac{3}{2}\ln|x-1| - \frac{3}{4}\ln(x^2+1) - \frac{1}{2}\tan^{-1}x + C \qquad (3\text{ pts})$$

(F) (12-points) $\int \frac{x^2}{(4-x^2)^{3/2}}\,dx$

let $x = 2\sin\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow dx = 2\cos\theta\, d\theta$ (2 pts)



(1 pt)

$$I = \int \frac{4\sin^2\theta}{(4\cos^2\theta)^{3/2}} \cdot 2\cos\theta\, d\theta \quad \text{(3 pts)}$$

$$= \int \frac{4\sin^2\theta}{8\cos^3\theta} \cdot 2\cos\theta\, d\theta$$

$$= \int \frac{\sin^2\theta}{\cos^2\theta}\, d\theta = \int \tan^2\theta\, d\theta \quad \text{(2 pts)}$$

$$= \int (\sec^2\theta - 1)\, d\theta = \tan\theta - \theta + C \quad \text{(2 pts)}$$

$$= \frac{x}{\sqrt{4-x^2}} - \sin^{-1}\left(\frac{x}{2}\right) + C \quad \text{(2 pts)}$$

(Q7) (5-points) Evaluate $\int \sin(3x)\cos(2x)\,dx$

$$I = \frac{1}{2}\int \left[\sin(3x-2x) + \sin(3x+2x)\right] dx \quad \text{(2 pts)}$$

$$= \frac{1}{2}\int (\sin x + \sin 5x)\, dx \quad \text{(1 pt)}$$

$$= -\frac{1}{2}\cos x - \frac{1}{10}\cos(5x) + C \quad \text{(2 pts)}$$