KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
DHAHRAN, SAUDI ARABIA

STAT 319: PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS

Major Examination No. II, Term 121

Time: 18: 00- 19:30 PM, Tuesday, 20th November, 2012

Please Check/circle the name of your instructor; Write section number and serial number in the top left corner of this booklet or below:

Section # /Serial # ID#

Name in Capital Letters: ______________________________

Instructors: □ Anabosi □Joarder
□ Malik

Coordinator: Anwar Joarder

You are allowed to use electronic calculators and other reasonable writing accessories that help write the exam. Try to define events, formulate problem and solve. See example below.

Do not keep your **mobile** with you during the exam, turn off your mobile and leave it aside.

No	Marks	Marks Obtained	Strengths and Weakness Observed
1	4		
2	5		
3	8		
4	14		
5	4		
6	3		
Total	38		

Kindly report grade to the coordinator out of 15 so that students know his precise standing. You may assign fractional marks if deemed necessary.

1..[3+1=4 Marks] A manufacturing company uses an acceptance plan on production items before they are shipped. The plan is a two-stage one. Boxes of 25 are prepared for shipment and a sample of 3 is tested for defectives. If any defectives are found, the entire box is sent back for 100% retesting. If no defectives are found, the box is shipped.

a.. What is the probability that a box containing 4 defectives will be shipped?
b.. How many elements do you have in your sample space?

a. P(To be shipped) = P(no defectives among the sample of 3)

If X: # of defective items in the sample of 3 $\Rightarrow X: HG(N, a, n)$

where $N = 25$, $a = 4$, $n = 3$ $\Rightarrow f(x) = \dfrac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}}$

$$\Rightarrow P(x=0) = \frac{\binom{4}{0}\binom{21}{3}}{\binom{25}{3}} = \frac{1330}{2300} = \frac{133}{230} = \boxed{0.5783}$$

(1) (1) (1)

b. # of elements in the sample space of the experiment of selecting any 3 out of the box of 25 is $\binom{N}{n} = \binom{25}{3} = \boxed{2300}$

(1)

Another solution for part(a):

$$P(D_1' D_2' D_3') = \frac{21}{25} \cdot \frac{20}{24} \cdot \frac{19}{23} = \frac{7980}{13800} = \frac{133}{230} = 0.5783$$

(1) (1) (1)

2.. [3 +2 = 5 Marks] If an instructor is visited, one an average, by 1 student per day with the following probability function:

$$f(x) = \frac{0.367879441}{x!}, \quad x = 0, 1, 2, \cdots$$

a. What is the probability that more than 2 students will visit the instructor every day?
b.. Determine the average of the number of students that visit the instructor every day.

a. $P(X>2) = 1 - P(X \le 2) = 1 - [f(0) + f(1) + f(2)]$, where $e^{-1} = 0.36787944$

$= 1 - e^{-1}\left[\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!}\right] = 1 - (2.5)e^{-1} = \boxed{0.0803}$

(1) (1) (1)

b. The average number of students $= \lambda t = 1(1) = \boxed{1}$ student

(1) (1)

3. [2+3+3=8 marks] An electronic switching device occasionally malfunctions and may need to be replaced. It is known that the device is satisfactory if it makes, on the average, no more than 0.2 error per hour. A particular 5-hour period is chosen as a "test" on the device. If no more than 1 error occurs, the device is considered satisfactory.

a.. What is the probability that a satisfactory device will be considered unsatisfactory on the basis of the test?

Denote this probability by p.

b.. If a sample of 7 devices is randomly selected, what is the probability that at least 2 of them will be considered unsatisfactory on the basis of the test?

c.. If the devices are tested one by one, what is the probability that the first unsatisfactory device will be found on the 5th test?

$\lambda = 0.2$/hour, $t = 5$ hours, x: # of errors in 5-hour period

$$\Rightarrow f(x) = \frac{e^{-\lambda t}(\lambda t)^x}{x!} = \frac{e^{-1}1^x}{x!} = \frac{e^{-1}}{x!}, \quad x = 0,1,2,\ldots$$

a. $P(X>1 \mid \lambda t = 1) = 1 - P(X \le 1) = 1 - \{f(0) + f(1)\}$ (1)

$$= 1 - e^{-1}\left[\frac{1}{0!} + \frac{1}{1!}\right] = 1 - e^{-1}(2) =$$

$$= 1 - \frac{2}{e} = \boxed{0.2642} = p$$ (1)

b. Y: # of unsatisfactory devices out of 7. $n = 7$, $p = 0.2642$

$Y: B(7, 0.2642) \Rightarrow f(y) = \binom{7}{y}(0.2642)^y(0.7358)^{7-y}, \quad y = 0,1,\ldots$ (1)

$P(Y \ge 2) = 1 - P(Y \le 1) = 1 - [f(0) + f(1)]$ (1)

$$= 1 - \left[\binom{7}{0}(0.2642)^0(0.7358)^7 + \binom{7}{1}(0.2642)^1(0.7358)^6\right]$$

$$= 1 - [0.1168 + 0.2935]$$

$$= 1 - 0.4102 = \boxed{0.5898}$$ (1)

c. X: # of tests to the first unstatisfactory device.

(1) $X: G(0.2642) \Rightarrow f(x) = q^{x-1}p, \quad x = 1,2,3,\ldots$

$P(X=5) = f(5) = q^4 \cdot p$ (1)

$$= (0.7358)^4 \cdot (0.2642) = \boxed{0.0774}$$ (1)

4.. [2+1+4+3+4=14] The lifetime (X) of a television picture tube is a normal random variable with mean 8.2 years and standard deviation 1.4 years.

a.. Write out the events A and B in terms of X where A is the event that the picture tube will survive more than 5 years and B is the event that the picture tube will survive more than 10 years.

b. Write out the event that tubes last between 5 and 10 years in terms of A and B (use complements, union or intersection etc)

c. What percentage of such tubes last between 5 and 10 years?

d. If a tube survives more than 5 years, what is the probability that it will survive more than 10 years? Use the notations A and B.

e.. Find the minimum life time for the best (surviving longer) 10% picture tubes?

X: life time of a TV picture in years. $X: N(8.2, (1.4)^2)$

a. $A: X>5 = (5,\infty)$ (1), $B = X>10 = (10,\infty)$ (1)

b. $A\cap B' = (5,10)$ (1) $= 5<X<10$

c. $P(A\cap B') = P(5<X<10)$ (1) $= P(X<10) - P(X<5)$

$\stackrel{(1)}{=} P\left(\frac{X-\mu}{\sigma} < \frac{10-8.2}{1.4}\right) - P\left(\frac{X-\mu}{\sigma} < \frac{5-8.2}{1.4}\right)$

$\stackrel{(1)}{=} F\left(\frac{9}{7} = 1.29\right) - F\left(-\frac{16}{7} = -2.29\right)$

$= 0.9015 - 0.0110 = \boxed{0.8905}$ (1)

d. $P(B|A) \stackrel{(1)}{=} \frac{P(A\cap B)}{P(A)} = \frac{P(B)}{P(A)} = \frac{P(X>10)}{P(X>5)}$ (1)

$= \frac{P(Z>1.29)}{P(Z>-2.29)} = \frac{F(-1.29)}{F(2.29)} = \frac{0.0985}{0.9890}$

$= \boxed{0.0996}$ (1)

e. $P(X>P_{90}) = 0.1$ or $P(X<P_{90}) \stackrel{(1)}{=} 0.9 \Rightarrow$

$\Rightarrow P\left(\frac{X-\mu}{\sigma} < \frac{P_{90}-8.2}{1.4}\right) = 0.9$ (1) $\Rightarrow$

$\Rightarrow \frac{P_{90}-8.2}{1.4} = 1.28$ (1) $\Rightarrow P_{90} = 8.2 + 1.28(1.4)$

$\Rightarrow P_{90} = \boxed{9.992}$ years

5..[4 Marks] Adding graphite to iron can improve its ductile qualities, If the measurements of the diameter of graphite spheres within an iron matrix can be modeled as a normal distribution having standard deviation 0.16. What is the probability that the mean of a sample of size 36 will differ from the population mean by more than 0.06?

X : Diameter of graphite sphere -

$X : N(\mu, (0.16)^2)$, $n = 36$ $\Rightarrow$

$\bar{X} : N\left(\mu, \frac{(0.16)^2}{36}\right) \rightarrow \mu$, $\sigma = \frac{0.16}{6} = 0.0267$ 1

$P(|\bar{X} - \mu| > 0.06) = P\left(\left|\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}\right| > \frac{0.06}{0.0267}\right)$ 1

$= P(|Z| > 2.25)$ 1

$= P(Z > 2.25 \text{ or } Z < -2.25)$

$= 2F(-2.25)$

$\doteq 2(0.0122) = \boxed{0.0244}$ 1

6. [3 Marks] The length of service (years) provided by a certain model of laptop computer is a random variable has the following density function:

$f(x) = 0.2e^{-0.2x}$ for $x \geq 0$, and $f(x) = 0$, $x < 0$.

Suppose that only 5% laptops have satisfactory service for more than k years. Determine k and name it in percentile.

X: Length of service in years.

$X : Exp(0.2) \Rightarrow \lambda = 0.2 \Rightarrow f(x) = 0.2e^{-0.2x}$

1 $P(X > k) = e^{-\lambda k} = e^{-0.2k} = 0.05$

$k = P_{95} = 95^{th}$ percentile 1

$\Rightarrow e^{-0.2 P_{95}} = 0.05$

$\Rightarrow -0.2 P_{95} = \ln(0.05)$

1

$\Rightarrow P_{95} = -5\ln(0.05) = \boxed{14.9787}$ years