KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DEPARTMENT OF MATHEMATICS & STATISTICS

DHAHRAN, SAUDI ARABIA

STAT 201: INTRODUCTORY STATISTICS

Semester 121
Major Exam Two
Saturday November 17, 2012

Name: Student ID#: Serial #:

Directions:

1) You must **show all work** to obtain full credit for questions on this exam.
2) **DO NOT round** your answers at each step. Round answers only if necessary at **your final step to 4 decimal places**.
3) You are allowed to use electronic calculators and other reasonable writing accessories that help write the exam. Try to define events, formulate problem and solve.
4) Do not keep your mobile with you during the exam, turn off your mobile and leave it aside

Question No	Full Marks	Marks Obtained
Q1	13	
Q2	18	
Q3	13	
Q4	13	
Q5	13	
Total	70	

Question One (13 points)

A certain type of component is packed in lots of four. Let X represents the number of properly functioning components in a randomly chosen lot. Assume that the probability that exactly x components function is proportional x; in other words, assume that the probability mass function of X is given by

$$P\left(X = x\right) = \begin{cases} c\ x & , x = 1,2,3,or\ 4 \\ 0 & otherwise \end{cases}$$

Where C is a constant

1. (***5 points***) Find the value of the constant C so that $P\left(X = x\right)$ is probability mass function. Then find the expected value and standard deviation of the number of properly functioning components

2. (***1 points***) Find probability that more than two properly functioning components.

3. *(**7 points**)* If a sample of size 36 components, find the probability that the average number of the components function is less than 3.1

Question Two (18 points)

The reading given by a thermometer calibrated in ice water (actual temperature 0°C) is a random variable with probability density function

$$f(x)=\begin{cases} k\left(1-x^2\right) & , -1<x<1 \\ 0 & otherwise \end{cases}$$

Where k is a constant

1. (***2 points***) Find the value of k

2. (***1 point***) What is the probability that the thermometer reads 0°C?

3. (***2 points***) What is the probability that the reading is within 0.25°C of the actual temperature?

4. (***3 points***) What is the median reading?

5. (***10 points***) If a sample of size 49 thermometers selected at random, find the probability that the average reading is more than 0. 15°C of the actual temperature

Question Three (13 points)

An insurance company offers a discount to home owners who install smoke detectors in their homes. A company representative claims that 80% or more of policyholders have smoke detectors.

1. (***2 points***) If you keep selecting policyholders until the first policyholders have smoke detectors, what is the probability that the first policyholders have smoke detectors is the 6^{th} one.

2. (***2 points***) If you draw a random sample of four policyholders from 30 policyholders, what is the probability that the first two policyholders have smoke detectors?

3. (***2 points***) If you draw a random sample of four policyholders from 30 policyholders, what is the probability that the two policyholders have smoke detectors?

4. (***2 points***) If you draw a random sample ***with replacement*** of four policyholders from 30 policyholders, what is the probability that the two policyholders have smoke detectors?

5. (***5 points***) Approximate, by normal distribution, the probability that in a sample of 30 policyholders, more than 25 policyholders have smoke detectors?

Question Four (13 points)

Researchers studying the effects of a new diet found that the weight loss over a one-month period by those on the diet was normally distributed with a mean of 9 pounds and a standard deviation of 3 pounds.

1. (***2 points***) What proportion of the dieters gained weight?

2. (***2 points***) If a dieter is selected at random, what is the probability that the dieter lost more 12 pounds?

3. (***2 points***) If 2950 dieters lost more than 12 pounds, how many persons took the test?

4. (***7 points***) If a sample of size 9 dieters selected at random, what is the probability that the group of dieters lose more than 85 pounds?

Question Five (13 points)

1. (***6 points***) A Poisson distribution is such that the probability that the random variable takes the value 1 is the same as the probability that it takes the value 2. What is the probability?

2. In a particular year, 1000 policies gave rise to 140 claims.
   a. (***2 points***) Find the probability that a particular policyholder makes no claim in nine months.

   b. (***5 points***) Find the probability that, during a given year, two independent policyholders have exactly one claim between them