King Fahd University of Petroleum & Minerals

Department of Mathematics & Statistics

Math 692 (121)
Fractional Differential Equations

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HW # 1

(1) Derive formula (1.99)

(2) Show that $y(t) = E_{2,1}(t^2)$ satisfies $y'' - y = 0$.

(3) Use the `mlf` Matlab function to plot $E_2(t^2)$, $t \in [-2, 2]$.

HW # 2

(1) Let $f(t) = t^{\sqrt{2}}$. Does $f \in AC^2[0, 2]$? Justify.

(2) Let $f(t) = |t|$. Does $f \in AC[0, 2]$? Justify.

(3) Show that

$$\lim_{\substack{h\to 0\\ nh=t-a}} \Delta_h^{-2} f(t) = I_a^2 f(t), \qquad t > a.$$

HW # 3

(1) Let $f \in C[0, b]$ be such that $I^p f$, $p > 0$ is constant on $[0, b]$. Describe the function f.

(2) Find the general solution $u \in C[a, b]$ that satisfies $D^p u(t) = 0$, $0 < p < 1$, and $t \in (0, b)$.

Exam 1

(1) Show that if $f \in C^1[a,b]$ then $I_a^\alpha D_f^\alpha(t)$, $0 < \alpha < 1$, is defined and it is equal to $f(t)$ for all $t \in [a,b]$.

(2) Consider the equation

$$D_0^{3/2} u(t) = \sqrt{t}, \qquad t > 0.$$

(a) Find the general solution.

(b) Find the general solution which bounded at $t = 0$.

(3) Consider the function

$$f(t) = \begin{cases} 1, & 0 \le t \le 2, \\ 0, & 2 < t < T. \end{cases}$$

Calculate $I_0^\alpha D_0^\alpha f(t)$ and $I_0^\alpha\, {}^C D_0^\alpha f(t)$, $\alpha > 0$.

(4) Calculate $D_1^{3/2}[t\,(t-1)^{1/4}]$.

(5) Show that if $f \in C^1[a,b]$ then

$$I_a^{1+\alpha} D f(t) = I_a^\alpha f(t) + \frac{f(a)}{\Gamma(\alpha+1)}\,(t-a)^\alpha.$$

Exam 2

(1) Consider the equation

$$D_{0+}^{5/2} y(x) - \lambda \frac{y(x)}{x} = f(x).$$

(a) Supplement this equation by the appropriate initial data when

(i) $D_{0+}^{5/2} = {}^{RL}D_{0+}^{5/2}$,

(ii) $D_{0+}^{5/2} = {}^{C}D_{0+}^{5/2}$.

(b) Solve the homogeneous problems by reduction and successive approximation.

(c) Solve the inhomogeneous problems.

(d) Determine 3 linearly independent solutions and find their Wronskian.

(e) How do we use the Wronskian to prove that these 3 solutions are linearly independent.

(2) Consider the equation

$$2\, D_{0+}^{3/4} y(x) + \sqrt{3}\, D_{0+}^{1/2} y(x) + \pi\, y(x) = \sin x.$$

where D_{0+}^α is the RL-derivative.

(a) Solve the homogeneous equation using the Laplace transform.

(b) Solve the non-homogeneous equation.

Final Exam

(1) Solve the problem

$$D_0^\alpha f(t) = 2, \qquad 1 < \alpha < 2, \quad t > 0,$$

$$I_0^{2-\alpha} f(0) = 1, \qquad D_0^{\alpha-1} f(0) = 0.$$

(2) Let

$$f(t) = \begin{cases} 0, & 0 \le t \le 3, \\ \frac{1}{\Gamma(\alpha+1)}(t-3)^\alpha, & t > 3, \end{cases} \qquad \alpha > 0.$$

Calculate $D_0^\alpha f$ and $I_0^\alpha D_0^\alpha f$ for $t > 0$.

(3) Use Laplace transform to solve the two equations:

$$D_0^{1/2} u(t) + 2u(t) = 0, \qquad \text{and} \qquad {}^C D_0^{1/2} u(t) + 2u(t) = 0,$$

$t > 0$. Comment on the behavior of the solution of the two equations as $t \to 0^+$.

(4) Use the power series expansions to show that

$$I_0^\alpha e^{at} = t^\alpha E_{1,1+\alpha}(at), \qquad 0 < \alpha < 1.$$

(5) Solve the equation

$$I_0^\alpha u(t) + u(t) = \sqrt{2}, \qquad 0 < \alpha < 1, \qquad t > 0.$$

(6) Find a particular solution for the equation

$$x^{5/4} D_0^{5/4} u(x) + x^{1/4} D_0^{1/4} u(x) = f(x), \qquad x > 0.$$