KFUPM/ Department of Mathematics and Statistics ID#
MATH 460 / Test 2/ 121 Name:

Q1. Let P be the matrix $\frac{aa^T}{a^Ta}$ where a is a vector.

a) Show that $P^2 = P$.
b) Is P invertible? Why?
c) Show that the trace of P always equals one.

Q2. Let W be a subspace of R^3 spanned by the set S = $\{x_1, x_2, x_3\}$, where

$x_1^T = [\sqrt{2} \quad 0 \quad \sqrt{2}]$, $x_2^T = [\sqrt{2} \quad 1 \quad \sqrt{2}]$, $x_3^T = [\sqrt{2} \quad -\sqrt{2} \quad 0]$

a) Use Gram-Schmidt to find an orthonormal basis for W.
b) Find the QR-factorization of the matrix A = $[x_1 \quad x_2 \quad x_3]$
c) Use part (b) to solve the system Ax=b where $b^T = [1 \quad 0 \quad -1]$

Q3. A small company has been in business for three years and has recorded annual profits (in thousands of dollars) as follows

Year	1	2	3
Sales	7	4	3

Assuming that there is a linear trend in the declining profits, predict the year and the month in which the company begins to lose money.

Q4. Find the next Legendre Polynomial – a cubic orthogonal to 1, x, and $x^2 - \frac{1}{3}$ over the interval [-1, 1].

Q5. Find the pivots (without using the elimination) of the matrix

$$A=\begin{bmatrix} 2 & 1 & 2 \\ 4 & 5 & 0 \\ 2 & 7 & 0 \end{bmatrix}.$$

Q6. If A= $\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ find A^{100} $by\ diagonalizing\ A$.

Q7. (a) Let A be a symmetric matrix. Show that all eigenvalues of A are real.

(b) Let v ϵ C^n. Show that if $v^*v = 1$, then $I - 2vv^*$ is hermitian and unitary.

Q8. Suppose T is a 3 x 3 upper triangular matrix with entries t_{ij} . Compare the entries of T$T^H$and T^HT, and show that if they are equal then T must be diagonal.

Q9. Find a unitary U and a triangular T so that $U^{-1}AU = T$, for $A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

Q10. Find the Jordan form J and the matrix M for $A=\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. What is the solution to $\frac{du}{dt} = Au$ and what is e^{At}?

Q11. Label each of the following statements as True or False.

(a) If the vectors x and y are orthogonal and P is a projection matrix, then Px and Py are also orthogonal.

(b) If all eigenvalues of a real matrix are equal to 1 then the matrix must be orthogonal.

(c) The sum of two unitary matrices must be unitary

(d) If N is a unitary matrix, then there exists a unitary matrix U such that $U^{-1}NU$ is a diagonal matrix

(e) If N is normal, then $\|Nx\| = \|N^H x\|$ for every vector x.