KFUPM/ Department of Mathematics and Statistics — ID#
MATH 460 / Test 1/ 121 — Name:

---

1. [8 pts] Factor A= $\begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix}$ into the decomposition LU, and use it to solve the system Ax=b, where $b = [2\ 2\ 5]^T$.

2. [6pts] Consider the system

   x+by =0

   x- 2y – z=0

   y+z=0

   Which number b leads to a row exchange? Which b leads to a missing pivot? In that singular case, find a nonzero solution x, y, z.

3. [10 pts] Given that A is invertible and $A = L_1 D_1 U_1$ and A=$L_2 D_2 U_2$.
   (a) Derive the equation $L_1{}^{-1} L_2 D_2 = D_1 U_1 U_2{}^{-1}$ and explain why one side is lower triangular and the other side is upper triangular.
   (b) Compare the main diagonals in that equation, and then compare the off diagonals.
   (c) Prove that $L_1 = L_2$, $D_1 = D_2$ and $U_1 = U_2$.
   (d) Use (c) to show that if A is symmetric, then the factorization becomes $A = LDL^T$.

4. [10 pts] For the system Ax=b where $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$ and b=$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$
   (a) Reduce [A b] to [U c}, to reach the echelon matrix U.
   (b) Find the condition on b_1, b_2 and b_3 to have a solution.
   (c) Describe the column space of A.
   (d) Describe the null space of A.
   (e) Find a particular solution when b=(-6, 6, -6) and the complete solution $x_p + x_n$.

5. [4 pts] If A is a 64 by 17 matrix of rank 11, how many independent vectors satisfy Ax=0? How many independent vectors satisfy $A^T y = 0$?

6. [6 pts] Find a basis for the row space and a basis for the left null space of the matrix

$$A=\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}.$$

7. [6 pts] Compute all possible right inverses of the matrix $A=\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \end{bmatrix}$. Do we have left inverses for A.

8. [5 pts] Let L be a transformation from R^2 to R^2 such that it is the composition of the following: A reflection in the line through O at 60 degrees to the positive x-axis, followed by counterclockwise rotation through 30 degrees, followed by projection onto the x-axis. Find the matrix A in the standard basis of R^2.

9. [5 pts] Suppose A=$\begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$. Find a nonzero matrix M such that T(M)=AM is zero. Can we find a matrix M such that T(M)=I where I is the identity matrix. Find the null space of T.