King Fahd University of Petroleum and Minerals
Department of Mathematics & Statistics
Math 425 Final Exam Fall 2012(121)

ID#:________________ NAME:________________________________

Total Score(out of 105)#________________ Time allowed: 150 minuts
NO CREDITS WILL BE GIVEN FOR ANSWER WITHOUT EXPLANATION.

PART ONE: (35 POINTS)

(1) Consider the graph $G = K_{2,3,2}$. Answer each of the following. If your answer is no, explain why; and if yes, support it by construction or calculation.

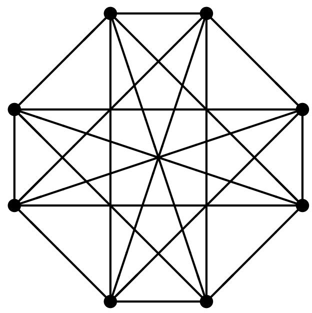
1. Is G Eulerian graph?
2. Is G Hamiltonian graph?
3. Is G planar graph?
4. Find the number of cut vertices and bridges of G.
5. Find the crossing number $cr(G)$.

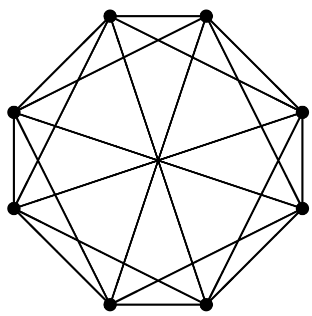
(2) Consider the complete graph $G = K_4$.

1. Find the number of spanning trees of G.
2. Let $A = (a_{ij})$ be the adjacency matrix of G. what is the graph theoretical meaning of:
   (a) the entry $a_{23}^{(2)}$ in A^2.
   (b) $\frac{1}{3}\sum_{i} a_{ij}^{(3)}$.

(3)

1. Prove that if G is a graph of order $n \geq 3$ and size $m \geq \binom{n-1}{2} + 2$, then G is Hamiltonian.
2. The following two graphs are isomorphic. Why?





PART TWO: (70 POINTS) (SOLVE ONLY TWO PROBLEMS FROM QUESTIONS 1,2,3,4 AND ALL 5,6 & 7.)

1. (a) Show that the extremal number $ex(n; C_n) = \binom{n-1}{2} + 1$.

   (b) Prove that if G is maximal outerplanar, then G has $n-2$ interior regions, where n is the order of G.

2. (a) Define the Moore bound $M(r, g)$.

   (b) Show that if G is an (r, g)-graph of order n, then $n \geq M(r, g)$.

3. (a) Find $n(3, 5)$, the smallest order of an $(3, 5)$-graph.

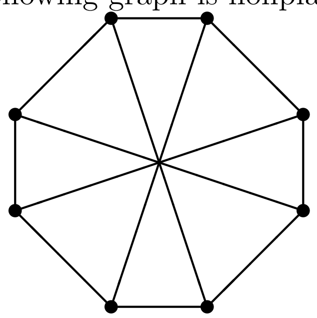
   (b) Show that the Peterson graph is the unique 5-cage.

4. (a) Let G be a connected planar graph with $n \geq 3$ vertices, m edges and girth $g = 5$. Show that $q \leq \frac{5}{3}(n-2)$.

   (b) Use (a) to show that the Peterson graph is non-planar.

5. (a) State Kuratowski's Theorem and Wagner's Theorem.

   (b) Use either Kuratowski's Theorem or Wagner's Theorem to prove that the following graph is nonplanar.



6. (a) Define (i) a vertex transitive graph. (ii) Similar vertices.

   (b) State Frucht's Theorem.

   (c) Show that every n-cycle is a Cayley graph.

   (d) How many distinct labeling (from a fixed set of labels) are there for (i) C_n ($n \geq 3$). (ii) P_n ($n \geq 2$). (iii) $K_{1,n}$ ($n \geq 2$)

7. (a) Define (i) A flow in a network N. (ii) A cut in a network N. (iii) A tournament. (iv) a strongly connected digraph.

   (b) State (i) the Max-Flow Min-Cut Theorem. (ii) Robbins' Theorem.

   (c) Prove that a tournament contains a vertex from which every other vertex can be reached by a directed path of length at most 2.

   (d) Prove or disprove: If every vertex of a tournament T belongs to a cycle in T, then T is strong.

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