King Fahd University of Petroleum and Minerals
Department of Mathematics & Statistics
Math 425 Exam III Fall 2012(121)

ID#:_____ NAME:_____

Total Score#_____
NO CREDITS WILL BE GIVEN FOR ANSWER WITHOUT EXPLANATION.

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(1) (a) Let N be a network with underlying digraph D, source u and sink v. For a set X of vertices of D with $u \in X$ and $v \in \overline{X}$ and a flow f defined on N, prove that $f^+(X) - f^-(X) = f(X, \overline{X}) - f(\overline{X}, X)$.
(b) Find the maximum number of internally disjoint $u - v$ paths in the digraph D shown below.

(2) Describe the automorphism groups of each of the following and then find their orbits. (i) $K_1 + \overline{K}_{1,n}$ (ii) $K_4 - x$, where $x = \{13\}$ (iii) D_1 :

(3) (a) Construct the cayley color graph of the cyclic group of order 3, $\mathbb{Z}_3 = \{0, 1, 2\}$, when the generating set Δ has two elements.
(b) Construct a graph G whose group $\Gamma(G) \cong \mathbb{Z}_3$.
(c) Show that every n-cycle is a cayley graph.

(4) (a) Show that if G is a plane graph with n vertices, m edges and r regions, then $n - m + r = 1 + k(G)$.
(b) Prove that there exists only one 4-regular maximal planar graph.
(c) Show that every graph G of order $n \geq 6$ that contains three spanning trees T_1, T_2 and T_3 such that every edge of G belongs to exactly one of these three trees is nonplanar.