King Fahd University of Petroleum and Minerals
Department of Mathematics & Statistics
Math 425 Exam II Fall 2012(121)

ID#:⎯⎯⎯⎯⎯⎯⎯⎯   NAME:⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Total Score#⎯⎯⎯⎯⎯⎯⎯⎯⎯
NO CREDITS WILL BE GIVEN FOR ANSWER WITHOUT EXPLANATION.

(1) (a) Find a graph G of diameter 3 for which $\lambda(G) \neq \delta(G)$.

(b) If $H = G + \{v\}$, where G is n-connected, then H is $(n+1)$-connected.

(c) Let G be a graph with degree sequence $d_1, d_2, ..., d_n$, where $d_1 \leq d_2 \leq ... \leq d_n$. Define $H = G + K_1$. Determine $\lambda(H)$.

(2) (a) State (i) Menger's theorem. (ii) Whitney's theorem.
(b) Let G be a graph on 8 vertices. (i) If G is 2-connected, how many edges must G have? (ii) If G is 3-connected, how many edges must G have?

(3) For what value(s) of n are the following graphs Hamiltonians and for what value(s) of n are they Eulerians?

(a) K_n (b) $K_{n,n}$ (c) $K_{n,n,n}$ (d) $K_{n,2n,3n}$ (e) $K_{n,2n,3n+1}$

(4) Prove any THREE statements of the following:

(a) Every two vertices in a nontrivial regular tournament lie on a 3-cycle. If regular is replaced by strong, is the statement still correct?

(b) A graph G has an Eulerian orientation if and only if G is Eulerian.

(c) A tournament is transitive if and only if it is acyclic.

(d) If $s_1, s_2, ..., s_n$ is the score sequence of a tournament then $\sum_{i=1}^{n} s_i^2 = \sum_{i=1}^{n} (n-1-s_i)^2$

Dr. M. R. Alfuraidan