King Fahd University of Petroleum and Minerals
Department of Mathematics & Statistics
Math 425 Exam1 Fall 2012(121)

ID#:________________ NAME:________________________________

Total Score#________________

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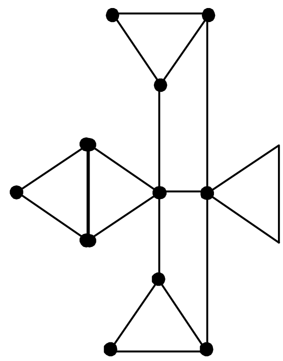
(1) Consider the graph G in the figure: (20pts)

G: 

(a) Is the graph G bipartite? If yes, give a bipartition. If not, explain why not?
(b) How many bridges does G have? List them.
(c) How many cut-vertices does G have? List them, then find their degrees in $\overline{G}$.
(d) List all values of n for which K_n is a subgraph of G?
(e) Find a subgraph that is not an induced subgraph of G?
(f) How many blocks does G have? Draw them.
(g) Find a spanning subgraph having a minimum number of edges?

(2) Determine if the following statements are TRUE or FALSE. If a statement is true, sketch the proof; if it is false, give a counterexample. (25pts)

(a) If G and H are two simple graphs with same degree-sequence, then they are isomorphic.

(b) If G is bipartite, then $\overline{G}$ is bipartite.

(c) If G is connected, then $\overline{G}$ is disconnected.

(d) If G is regular, then $\overline{G}$ is regular.

(e) If v is a cut-vertex of a simple graph G, then v is a cut-vertex of $\overline{G}$

(f) If G is a nontrivial connected graph such that each vertex is of even degree, then G has at least one circuit.

(g) The following two graphs are isomorphic.

(3) Find each of the following: (30pts)

(a) The maximum number of bridges in a tree T with n vertices.

(b) The maximum number of vertices in a graph G with 15 edges and 3 components.

(c) The minimum number of cut vertices in a tree T with n vertices.

(d) All trees T whose complement $\overline{T}$ is a also a tree.

(e) The number of labeled spanning forests in $\overline{K_{4,4}}$.

(f) The labeled tree having Prüfer sequence $(4, 5, 7, 2, 1, 1, 6, 6, 7)$.

(g) x and y (by using Havel-Hakimi Theorem) if $S : 7, 5, 3, x, 2, 2, 2, y, 1, 1, 1$ is a graphical degree sequence.

(4) Answer only 2 problems (15pts)

(a) Prove that a simple graph G with $2n$ vertices such that the degree of each vertex is at least n is connected.

(b) Show that a tree T has one and only one vertex in the center if and only if $diam(T) = 2\ rad(T)$.

(c) Prove Cayley's Tree Formula (for each positive integer n, there are n^{n-2} distinct labeled trees of order n having the same vertex set) as a corollary to the Matrix-Tree Theorem.

(d) Prove that every nontrivial connected graph contains at least two vertices that are not cut-vertices.

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