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MAJOR No. 2
MATH. 411-121

Name:

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Prob. 1

Let $f(x,y) = 2xy(x^2+y^2)^{-1/2}$ if $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Find the derivative of f at $(0,0)$ at any direction V.

Prob. 2

Let $n = 3$ and $L(x,y,z) = x + y + 2z$.

(a) What is the covector $A = (a_1, a_2, a_3)$ corresponding to L?

(b) Describe the set $\{(x,y,z) : L(x,y,z) = c\}$ and the intersection of this set with the plane $\{(x,y,z) : y = x\}$.

Prob. 3

(a) What are the additional conditions to a differentiable function f to be Lipschitzian. Prove the assertion.

(b) Let f be a differentiable function whose domain D is an open, connected set, such that $df(X) = 0$, for all $X \in D$. Prove that f is then a constant function.

Prob. 4

Let $f(x,y) = \Psi(ax+by)$, $a, b \in R$, $\Psi \in C^{(q)}$ in some open set containing 0. Find a simple form for its Taylor formula (Expansion) about $(0,0)$.

Prob. 5

(a) Let $f(x,y,z) = x^2 + y^2 - z^2$. Show that f has one critical point, which does not give a relative extremum.

(b) Let $f(x, y, z) = x^2 + 3y^2 + 2z^2 - 2xy + 2xz$. Find the minimum value of f. Justify your answer!.