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MATH. 411-121

Name:

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Prob. 1

Prove that $f(x,y) = \frac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}$ has a removable discontinuity at $(0,0)$.

Prob. 2

Let $g(x,y) = \frac{2xy}{x^2+y^2}$ on $R^2\backslash\{(0,0)\}$. Show that g has no limit at $(0,0)$.

Prob. 3

Suppose that $f : E \to F$ is a continuous mapping between 2 topological spaces. Is the inverse image of a closed set under f always closed? Justify your answer.

Prob. 4

(a) Give the definition of connectedness. (b) Assume that E is a topological space and $X \in E$. Prove that $\{X\}$ is connected.

Prob. 5

Let $f(x,y) = \frac{4xy^2}{x^2+y^2}$ on $R^2\backslash\{(0,0)\}$. For $X = (0,y)$ or $X = (x,0)$, $f(X) = 0$. Prove, using the ε-δ argument, that $\lim_{X\to 0} f(X) = 0$.

Prob. 6

Prove that $R\backslash Q$ is neither open nor closed.

Prob. 7

Prove that $A \subset B \Longrightarrow \bar{A} \subset \bar{B}$ and that $\overline{A \cup B} = \bar{A} \cup \bar{B}$ (where the bar is for the closure).