King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 302 Exam 2 Semester (121) November 21, 2012 Time: 08:00 - 10:00 pm

Name: ....

I.D: ....Section: ....

No calculators allowed. For all steps show calculations. *Each question carries **20** marks.*

Problem	Points
1	
2	
3	
4	
5	
Total	

Exercise 1. Let $\varphi(x,y) = xy^3$ and $\mathcal{C}$ be the curve given parametrically by:

$$x = 2\cos t,\ y = 2\sin t \text{ with } 0 \le t \le \frac{\pi}{2}.$$

Evaluate the line integral

$$I = \int_{\mathcal{C}} \varphi ds$$

of φ along $\mathcal{C}$ with respect to arc length.

Exercise 2. Let $\mathcal{C}$ be the positively oriented simple closed path given by

$$\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2 \cup \mathcal{C}_3,$$

where

- $\mathcal{C}_1$ is the portion of the graph of $y = x^3$ joining the points $(0,0)$ and $(1,1)$,
- $\mathcal{C}_2$ is the portion of the graph of $y = x^2$ joining the points $(1,1)$ and $(\frac{1}{2}, \frac{1}{4})$,
- $\mathcal{C}_3$ is the line segment joining $(\frac{1}{2}, \frac{1}{4})$ and $(0,0)$.

Verify Green's Theorem for the vector field $\vec{F}(x,y) = 4y\hat{\imath} + 7x\hat{\jmath}$ over the path $\mathcal{C}$.

.

Exercise 3. Use Stokes theorem to evaluate $\oint_{\mathcal{C}} \bar{F}.\vec{dr}$, where $\vec{F}(x,y,z) = -y^2\hat{\imath} + x\hat{\jmath} + z^2\hat{k}$ and the curve $\mathcal{C}$ is the trace of the cylinder $x^2 + y^2 = 4$ in the plane $x + y + z = 3$. Assume that $\mathcal{C}$ is oriented counter clock-wise as viewed from above.

Exercise 4. Evaluate the surface integral $\int\int_S G(x,y,z)dS$, where $G(x,y,z) = (x^2+y^2+z^2)^2$ and S is the portion of the cone $z = 4\sqrt{x^2+y^2}$, $y \geq 0$ and $0 \leq z \leq 4$.

Exercise 5. Consider the vector field $\vec{F}(x, y) = 2xy^3\hat{\imath} + (1 + 3x^2y^2)\hat{\jmath}$ show that

- $\vec{F}$ is conservative
- Find a potential $\phi(x, y)$ whose gradient is the vector field $\vec{F}$.
- Evaluate the integral $I = \displaystyle\int_{(1,4)}^{(3,1)} 2xy^3dx + (1 + 3x^2y^2)dy$.