King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 302 Exam I Semester (121)   October 04, 2012   Time: 08:00 - 10:00 pm

Name: ....

I.D: ....Section: ....

No calculators allowd. For all steps show calculations

Problem	Points
1	
2	
3	
4	
5	
6	
7	
Total	

Exercise 1. [**16 points**] **Let** $S = \{(x, y, z) \in \mathbb{R}^3 \mid 2x - y + 3z = 0\}$**.**

(i) **Show that** S **is a subspace of** $\mathbb{R}^3$**.**

(ii) **Find** $\dim(S)$ **(the dimension of** S**).**

Exercise 2. [**10 points**] **Write (if possible) the vector** $< 1, -3, 2 >$ **as a linear combination of the three vectors** $< 2, 0, 0 >, < 1, 1, 0 >, < 4, -2, 1 >$.

Exercise 3. [16 points] Consider the non-homogeneous system given below:

$$\begin{cases} x_1 & - & 2x_2 & + & x_3 & = & 2 \\ 3x_1 & - & x_2 & + & 2x_3 & = & 5 \\ 2x_1 & + & x_2 & + & x_3 & = & 3 \end{cases}$$

(i) Solve the system by using Gauss-Jordan elimination method.

(ii) Does this system have a unique solution?

Exercise 4. [16 points] Let

$$A = \begin{pmatrix} -2 & -2 & 7 \\ 4 & 3 & -12 \\ -1 & 0 & 2 \end{pmatrix}.$$

(a) **Find A^{-1}, (use Gauss-Jordan elimination method).**

(b) **Using inverse found in part (a), solve the system:**

$$\begin{cases} -2x - 2y + 7z = 0 \\ 4x + 3y - 12z = 1 \\ -x + 2z = 2 \end{cases}$$

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Exercise 5. [16 points] Let

$$A = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 1 & 1 & -1 \end{pmatrix}.$$

(a) **Find the eigenvalues of** A**.**

(b) **Find the Eigenvectors of** A**.**

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Exercise 6. [16 points] The Eigenvalues and the corresponding Eigenvectors of the symmetric matrix,

$$A = \begin{pmatrix} 1 & 4 \\ 4 & 16 \end{pmatrix}$$

are given by:

$$\lambda_1 = 0, K_1 = \begin{pmatrix} 4 \\ -1 \end{pmatrix} \; and \; \lambda_2 = 17, K_2 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

(a) **Check if the Eigenvectors are orthogonal?**

(b) **Construct the matrix P that diagonalizes A orthogonally.**

(c) **Give the diagonal matrix D.**

$d)$ **Show that $P^T AP = D$.**

Exercise 7. [**10 points**] **Find value of '**a**' for which the rank of the matrix given below is '**2**':**

$$A = \begin{pmatrix} 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \\ -3 & 6 & -1 & 1 & a \end{pmatrix}.$$