King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 302
Quiz4

Semester (121) December 17, 2012 Time: 7:00 - 8:00 pm

Name: .....

I.D: .....

Exercise 1. Let $u(x, y) = 4xy^3 - 4x^3y + x$. Show that u is a harmonic function. Find a conjugate harmonic v of u and form the corresponding analytic function $f(z) = u + iv$.

Exercise 2. Find all values of z satisfying the following equation:

$$e^{2z} + e^z + 1 = 0.$$

Exercise 3. Find un upper bound for the absolute value of the following contour integral:

$$\int_{\mathcal{C}} \frac{1}{z^2 - 2i} \, \mathrm{d}z,$$

where $\mathcal{C}$ is the right half circle $|z| = 6$ from $z = -6i$ to $z = 6i$.

Exercise 4. Let $\mathcal{C}$ be a a positively oriented simple closed contour enclosing the complex number z_0 and n be a positive integer. Show that

$$\oint_{\mathcal{C}} \frac{1}{(z-z_0)^n}\,\mathrm{d}z = \begin{cases} 2\pi i & \text{if } n = 1 \\ 0 & \text{if } n \neq 1 \end{cases}$$

Exercise 5. Let $\mathcal{C}$ be a positively oriented simple closed contour enclosing the complex number 0. Evaluate the integral

$$\oint_{\mathcal{C}} \frac{1}{z^4 e^{2z}} \, \mathrm{d}z.$$