King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Final Exam
The First Semester of 2012-2013 (121)

Time Allowed: 180 Minutes

Name: ______________________________ ID#: ____________________

Instructor: ______________________ Sec #: __________ Serial #: __________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		18
2		20
3		22
4		22
5		22
6		18
7		18
Total		140

Q:1 (18 points) Solve the initial vale problem using Laplace transform,

$$y'' - 2y' + 2y = \delta(t-1) \text{ with } y(0) = 1,\ y'(0) = 0.$$

Q:2(20 points) Consider the Sturm–Liouville problem

$$y'' - 2y' + \lambda y = 0 \text{ with } y(0) = 0,\ y(2) = 0.$$

(a) Find the eigenvalues and corresponding eigenfunctions.

(b) Write the equation in self–adjoint form and write the weight function.

Q:3 (22 points) Use separation of variables method to solve the problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0,\ 0 < r < 1,\ 0 < z < 2,$$

subject to the boundary conditions

$$\begin{aligned} u(1, z) &= 0,\ 0 < z < 2 \\ u(r, 0) &= 0,\ 0 < r < 1 \\ u(r, 2) &= 1,\ 0 < r < 1 \\ \text{solution } u(r, z) \text{ is bounded at } r &= 0. \end{aligned}$$

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Q:4 (22 points) Find the steady-state temperature $u(r,\theta)$ in a sphere of radius 1 by solving the problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta} = 0, \quad 0 < r < 1,\ 0 < \theta < \pi,$$

subject to the boundary condition

$$u\,(1,\theta) \;=\; 5\cos(\theta), \quad 0 < \theta < \pi.$$

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Q:5 (22 points) Use Laplace transform to solve the problem

$$4\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < 1,\ t > 0,$$

subject to the boundary and initial conditions

$$\begin{aligned} u(0,t) &= 0, \quad u(1,t) = 0, \quad t > 0, \\ u(x,0) &= 0, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = \sin(\pi x), \quad 0 < x < 1. \end{aligned}$$

Q:6 (18 points) Find Fourier integral representation of

$$f(x) = \begin{cases} 0, & x < -1 \\ 2, & -1 < x < 0 \\ -x + 2, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$$

Q:7 (18 points) Use Fourier sine transform to solve the problem

$$2\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < \infty,\ t > 0,$$

subject to the boundary and initial conditions

$$\begin{aligned} u(0,t) &= 2, \quad t > 0, \\ u(x,0) &= 0, \quad\quad 0 < x < \infty. \end{aligned}$$