**King Fahd University of Petroleum & Minerals**
**Department of Mathematics & Statistics**
**Math 301 Major Exam 1**
**The First Semester of 2011-2012 (121)**

**Time Allowed: 120 Minutes**

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Name: ______________________ ID#: ______________

Instructor: ______________________ Sec #: ________ Serial #: ________

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- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

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| Question # | Marks | Maximum Marks |
|---|---|---|
| 1 | | 14 |
| 2 | | 12 |
| 3 | | 14 |
| 4 | | 16 |
| 5 | | 12 |
| 6 | | 16 |
| 7 | | 16 |
| Total | | 100 |

**Q:1** (7+7 points) Given the position vector $\vec{r}(t) = 2\sqrt{2}\, t\ \hat{\imath} + e^{2t}\hat{\jmath} + e^{-2t}\hat{k}$ of a curve $C$:

**(a)** Find a unit tangent vector to the curve at $t = 0$.

**(b)** Find the length of the curve for $0 \leq t \leq 1$.

**Q:2** (6+6 points) Let $f(x, y, z) = xy^2 + 2x^2y + z^2$.

**(a)** Find the directional derivative of $f(x, y, z)$ at $(-1, 1, 2)$ in the direction of $\hat{i} - \hat{j} + 3\hat{k}$.

**(b)** Find the direction and value of maximum directional derivative of $f(x, y, z)$ at $(1, 0, 1)$.

**Q:3 (a)** (8 points) Evaluate the line integral $\int_C (x - 3y)dx + ydy$ along the path from O to B as shown in the figure:

**(b)** (6 points) Evaluate the line integral $\int_C x^2\, dx + (x+z)\, dy + y^2\, dz$, where the path $C$ is given by $x = t,\ y = t^2,\ z = -t,\ \ 0 \le t \le 1.$

**Q:4** (16 points) Determine if the vector field

$$\vec{F}(x,y) = (e^x \sin y - y\sin(xy))\ \hat{\imath} + (e^x \cos y - x\sin(xy))\ \hat{\jmath}$$

is a conservative field. If so, find a potential function $\phi(x,y)$ for $\vec{F}$ and evaluate the integral $\int_C \vec{F} \cdot d\vec{r}$, where $C$ is any path from $(1, \frac{\pi}{2})$ to $(2, \pi)$.

**Q:5** (12 points) Use Green's theorem to evaluate the line integral $\oint_C 3e^{-x^2}dx + 2\ln(1+x^2)dy$, where $C$ is the positively oriented triangle with vertices $(0,0)$, $(0,1)$ and $(1,1)$.

**Q:6** (16 points) Evaluate the surface integral $\int\int_S y^2 z dS$, where S that portion of the cone $z = \sqrt{x^2 + y^2}$ that lies between the planes $z = 1$ and $z = 2$.

**Q:7** (16 points) Use Stokes' theorem to evaluate the integral $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y, z) = 2yz\ \hat{i} + xy\ \hat{j} + xy\ \hat{k}$ and $C$ is the intersection of the plane $x + y + z = 2$ with the coordinate planes in the first octant oriented positively.