MATH 301.2 (Term 121)
Quiz 2 (Sects. 9.12, 9.13) Duration: 20mn

Name: **ID number:**

1.)(5pts)Use Green's theorem to evaluate the line integral $\oint_C \sqrt{x^2+y^2}dx+\sqrt{x^2+y^2}dy$ along the closed path C given by $y=0$, $x^2+y^2=1$, $y\le 0$.
2.)(5pts)Find the surface area of the sphere $x^2+y^2+z^2=16$ that lies within the cylinder $x^2+y^2=4y$.

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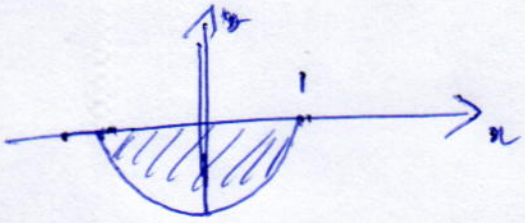
1) $\int_C F\,dr = \iint_R \left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dA$

$= \iint_R \left(\frac{x}{\sqrt{x^2+y^2}}-\frac{y}{\sqrt{x^2+y^2}}\right)dA$



$R=\{(r,\theta)/\ -\pi\le\theta\le 0,\ 0\le r\le 1\}$

$\int_C P\,dx+Q\,dy = \int_{-\pi}^{0}\int_0^1 \frac{r\cos\theta - r\sin\theta}{r}\,r\,dr\,d\theta$

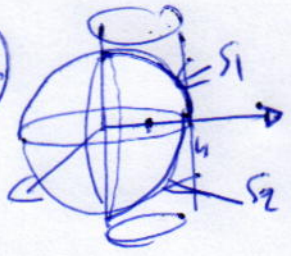
$= \left(\int_{-\pi}^{0}(\cos\theta-\sin\theta)\,d\theta\right)\left(\int_0^1 r\,dr\right)$

$= \frac{1}{2}\left[\sin\theta+\cos\theta\right]_{-\pi}^{0}$

$= \frac{1}{2}(2)$

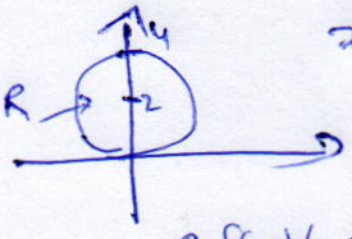
$= 1$

2)



$S_1: z=\sqrt{16-x^2-y^2}$

$S_2: z=-\sqrt{16-x^2-y^2}$



$x^2+y^2=4y$
$x^2+(y-2)^2=4$

$R=\{(r,\theta)/\ -\frac{\pi}{2}\le\theta\le\pi/2,\ 0\le r\le 4\sin\theta\}$

$A(S) = \iint_S dS = 2\iint_{S_1} ds + \iint_{S_2} ds = 2\iint_{S_1} ds$

$A(S) = 2\iint_R \frac{4}{\sqrt{16-x^2-y^2}}\,dA$

$= 8\int_{-\pi/2}^{\pi/2}\int_0^{4\sin\theta}\frac{r}{\sqrt{16-r^2}}\,dr\,d\theta$

$= 8\int_{-\pi/2}^{\pi/2}\left[-(16-r^2)^{1/2}\right]_0^{4\sin\theta}d\theta$

$= 8\int_{-\pi/2}^{\pi/2}\left(4-(16-16\sin^2\theta)^{1/2}\right)d\theta$

$= 32\int_{-\pi/2}^{\pi/2}(1-\cos\theta)\,d\theta = 32\left[\theta-\sin\theta\right]$

$= 32(\pi-2)$