MATH 301.3 (Term 121)
Quiz 2 (Sects. 9.12, 9.13) Duration: 20mn

Name: ID number:

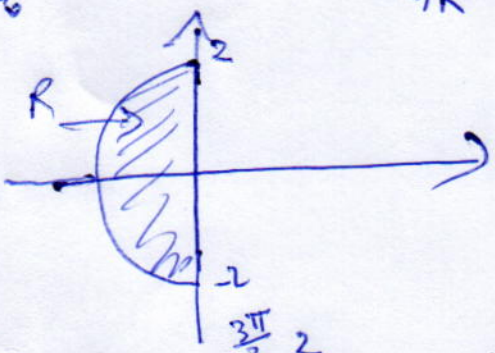
1.)(5pts)Use Green's theorem to evaluate $\oint_C (x^2+y^2)^{3/2}dx + (x^2+y^2)^{3/2}dy$ along the closed path C given by $x = 0$, $x^2 + y^2 = 4$, $x \le 0$.
2.)(5pts)Evaluate the surface integral $\iint_S (2x + 4z)ds$, where S is the portion of the plane $2x + 3y + 4z = 12$ in the first octant.

1.) $P = (x^2+y^2)^{3/2}$, $Q = (x^2+y^2)^{3/2}$

$P_x = \frac{3}{2}(2x)(x^2+y^2)^{\frac{1}{2}} = 3x(x^2+y^2)^{1/2}$

$P_y = 3y(x^2+y^2)^{1/2}$

$\oint_C (x^2+y^2)^{3/2}dx + (x^2+y^2)^{3/2}dy = \iint_R 3(x+y)(x^2+y^2)^{1/2}\,dA$



$\oint_C P\,dx + Q\,dy = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^2 3(r\cos\theta - r\sin\theta)\, r\, r\,dr\,d\theta$

$= \frac{3}{4}\left[r^4\right]_0^2 \left(\int_{\pi/2}^{3\pi/2} \cos\theta - \sin\theta\, d\theta\right)$

$= 12\left[\sin\theta + \cos\theta\right]_{\pi/2}^{3\pi/2}$

$= -24$

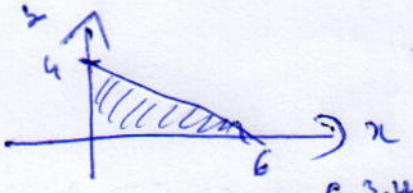
2.) $\iint_S (2x+4y)\,ds = \iint_R (2x + 12 - 2x - 3y)\sqrt{1+f_x^2+f_y^2}\,dA$

$z = f(x,y) = \frac{1}{4}(12 - 2x - 3y) = 3 - \frac{x}{2} - \frac{3}{4}y$

$f_x = -1/2$, $f_y = -3/4$

$\iint_S (2x+4y)\,ds = \iint_R 12 - 3y\,\sqrt{1+\frac{1}{4}+\frac{9}{16}}\,dA$

$= \frac{\sqrt{29}}{4}\iint_R (12-3y)\,dA$



$\iint_S (2x+4y)\,ds = \frac{\sqrt{29}}{4}\int_0^4 \int_0^{6-\frac{3}{2}y} (12-3y)\,dx\,dy$

$= \frac{\sqrt{29}}{4}\int_0^4 (12-3y)\left(6-\frac{3}{2}y\right)dx$

$= \frac{\sqrt{29}}{2\times 4}\left[\frac{-(12-3y)^3}{9}\right]_0^4$

$= \frac{\sqrt{29}}{36\times 2}\left(12^3 - 0\right)$

$= \frac{144}{2}\sqrt{29}$

$= 72\sqrt{29}$