King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 260 Exam II
Semester I, 2012 (121)

Name:	KEY	
ID :	KEY	**CODE A**
Serial no.:		
See	**(2:** Dr. Al-Shammari**)** **(3:** Dr. Al-Humidi**)** **(4:** Dr. Al-Humidi**)** **(5:** Dr. Mimouni**)** **(6:** Dr. Laradji**)**	

Q		Points
1		12
2		12
3		12
4		12
5		12
6		5
7		5
8		5
9		5
10		5
11		5
12		10 (5 each)
Total		**100**

(1) Solve the linear differential equation $xy' + y - x^3 = 0$ [12 points]

putting the equation in the standard form:

$$y' + \frac{1}{x}y = x^2 \qquad p(x) = \frac{1}{x} \text{ and } Q(x) = x^2$$

3pts

The integrating factor is

$$u = e^{\int p(x)dx} = e^{\int \frac{dx}{x}} = e^{\ln x} = x$$

3pts

Then:

$$y(x) = \frac{1}{u}\left[\int u\,Q(x)dx + c\right]$$

3pts

$$= \frac{1}{x}\left[\int x^3\,dx + c\right]$$

$$= \frac{1}{x}\left[\frac{1}{4}x^4 + c\right]$$

3pts

$$= \frac{1}{4}x^3 + c\frac{1}{x}$$

(2) Solve the separable differential equation $(1+x^2)dy + x(1+4y^2)dx = 0$.

[12 points]

Divide by $(1+x^4)(1+4y^2)$,

$$\frac{dy}{1+4y^2} + \frac{x\,dx}{1+x^4} = 0$$ 1

$$\Rightarrow \frac{dy}{1+4y^2} = -\frac{x}{1+x^4}\,dx$$ 5pts

$$\Rightarrow \int \frac{1}{1+(2y)^2}\,dy = -\int \frac{x}{1+x^4}\,dx$$ 1

$$\Rightarrow \frac{1}{2}\tan^{-1}(2y) = -\frac{1}{2}\ln(1+x^2) + c$$ 5pts

$$\Rightarrow \tan^{-1}(2y) + \ln(1+x^2) = C$$

(3) Consider the differential equation $(x^2+2x+3)dx+(x-xy)dy=0$ (*)

[12 points]

(a) show that the DE is not exact

$M = x^2 + 2x + 3$, $N = x - xy$

$M_y = 0$ 1 $\qquad N_x = 1 - y$ 1

$M_y \neq N_x$ so the equation is not exact.

(b) find an integration factor and convert the equation to exact

$$\frac{M_y - N_x}{N} = \frac{0-(1-y)}{x(1-y)} = -\frac{1}{x}$$ depends only on x 1

$$\mu(x) = e^{\int \frac{M_y - N_x}{N}} = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = -1/x$$ 2

Multiply the equation (*) by $-\frac{1}{x}$, we obtain

$$\underbrace{\left(-x-2-\frac{3}{x}\right)}_{M_2} dx \underbrace{-(1-y)}_{N_2} dy = 0$$ 1

(c) solve the DE.

$$(M_2)_y = 0 = (N_2)_x$$

$$F(x,y) = \int M(x,y)\,dx + g(y)$$ 2pts

$$= -\frac{x^2}{2} - 2x - 3\ln x + g(y)$$

$$\frac{\partial F}{\partial y} = N_2(x,y) \Rightarrow g'(y) = -(1-y)$$ so $\boxed{g(y) = \frac{y^2}{2} - y}$ 2pts

Thus the solution is given by

$$\boxed{-\frac{x^2}{2} - 2x - 3\ln x + \frac{1}{2}y^2 - y = c}$$ 2pts

(4) A bacteria culture has an initial population P_0. After one hour, the number of bacteria is found to be $\frac{3}{2}P_0$. If the rate of growth is proportional to the number of bacteria $P(t)$ present at time t. Determine the time necessary for the number of bacteria to triple. [12 points]

Let $P(t)$ be the number of bacteria at time t

then, $\frac{dP}{dt} = k\,P(t)$ with $P(0) = P_0$, $P(1) = \frac{3}{2}P_0$ 2

We need to find t s.t $P(t) = 3P_0$ 1

Solving the DE:

$\frac{dP}{P} = k\,dt \Rightarrow \ln P = kt + C_1$ 2

$\Rightarrow P(t) = Ce^{kt}$ $\quad P(0) = P_0 \Rightarrow C = P_0$ 1

$\Rightarrow P(t) = P_0 e^{kt}$ $\quad$ Now since $P(1) = \frac{3}{2}P_0$

$\Rightarrow P_0 e^{k} = \frac{3}{2}P_0 \Rightarrow e^{k} = \frac{3}{2} \Rightarrow k = \ln\left(\frac{3}{2}\right)$ 1

$\therefore \boxed{P(t) = P_0\, e^{t\ln\left(\frac{3}{2}\right)}}$ 3

$P(t) = 3P_0 \Rightarrow 3P_0 = P_0 e^{t\ln\left(\frac{3}{2}\right)} \Rightarrow 3 = e^{t\ln\frac{3}{2}}$

$\Rightarrow t\ln\frac{3}{2} = \ln 3 \Rightarrow t = \frac{\ln 3}{\ln\frac{3}{2}} \simeq 2.7\text{ h.}$

2

(5) Solve the differential equation $y^2 = \frac{(y')^3}{y''}$. [12 points]

put $y' = v$ 1 So that $y'' = v\frac{dv}{dy}$ 1

$$y^2 \cdot y'' = (y')^3$$

$$y^2 v \frac{dv}{dy} = v^3 \qquad (v \neq 0) \; 2$$

$$y^2 \frac{dv}{dy} = v^2$$

$$\Rightarrow \frac{dv}{v^2} = \frac{dy}{y^2} \; 2$$

$$\Rightarrow \frac{1}{v} = \frac{1}{y} + c_1 \; 3$$

So $dx = \left(\frac{1}{y} + c_1\right) dy$

and then $\boxed{x = \ln|y| + c_1 y + c_2}$ 3

(6) The Wronskian of $f(x) = e^{3x}$ and $g(x) = e^{-3x}$ is

$\boxed{\text{(a) } -6}$
(b) e^{6x}
(c) 0
(d) $2e^{3x}$
(e) e^{-3x} [5 points]

(7) The matrix $\begin{bmatrix} 1 & -3 & 2 & 2 \\ 0 & 3 & 6 & 0 \\ 3 & -6 & 0 & 6 \\ 2 & -3 & -2 & 4 \\ -2 & 9 & 2 & -4 \end{bmatrix}$ has rank

(a) 1
(b) 2
$\boxed{\text{(c) } 3}$
(d) 4
(e) 5 [5 points]

(8) Let $\mathbf{P}_2$ be the vector space of all polynomials of degree ≤ 2. Which one of the following sets spans $\mathbf{P}_2$?

(a) $\{1,\ x,\ x^3\}$
(b) $\{1,\ x^2,\ x^3\}$
(c) $\{1,\ x^2,\ 1+x^2\}$
(d) $\{1,\ x,\ 1+x\}$
(e) $\{1,\ 1+x,\ 1+x^2\}$ [5 points]

(9) Let $W = Span\{t, \cos t, \sin t\}$, a subspace of the space of all continuous functions. Which of the following sets is a basis for W?

(a) $\{t,\ \cos t,\ t+\cos t\}$
(b) $\{t,\ \sin t,\ t+\sin t\}$
(c) $\{\sin t,\ \cos t,\ \sin t+\cos t\}$
(d) $\{t,\ 2t+5\sin t,\ \sin t\}$
(e) $\{2t,\ 3\cos t,\ 5\sin t\}$ [5 points]

(10) The differential equation $x^2y' + y^2 = x^2y$ is

(a) Bernoulli
(b) Separable
(c) Homogeneous
(d) Linear
(e) Exact

[5 points]

(11) Which of the following statements is **False** for a vector space V with $\dim(V) = n$?

(a) Any set of vectors $\{w_1, w_2, ..., w_m\}$ with $m < n$ must be linearly independent.
(b) Every set of vectors $\{w_1, w_2, ..., w_m\}$ in V where $m > n$ is linearly dependent.
(c) If $w_1, w_2, ..., w_m$ form a basis for V, then $m = n$.
(d) If $w_1, w_2, ..., w_n$ are linearly independent vectors in V, then $w_1, w_2, ..., w_n$ form basis for V.
(e) If $Span\{w_1, w_2, ..., w_n\} = V$, then $w_1, w_2, ..., w_n$ form a basis for V.

[5 points]

(12) True or False (circle T or F)

a)	If A is 4×3 matrix and $\dim(NS(A))=1$, then $\dim(RS(A))=2$.	**T**	**F**
b)	If f_1, f_2 are differentiable functions on $(-\infty,\infty)$ and the Wronskian $W(f_1,f_2)=0$ for all $x\in(-\infty,\infty)$, then f_1 and f_2 are linearly dependent.	**T**	**F**

[10 points (5 each)]