King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 260 Exam 1
Semester I, 2012 (121)

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Q		Points	
1		10	5
2		12	
3		12	
4		10	
5		10	
6		10	
7		10	
8		7	
9		7	
10		7	
Total		**100**	

(1) Let A be the coefficient matrix of the following linear system

$$
\begin{aligned}
x_1 + 2x_2 + 3x_3 &= 6 \\
2x_1 - 3x_2 + 2x_3 &= 14 \\
3x_1 + x_2 - x_3 &= -2
\end{aligned}
$$

a) Find A^{-1} [10 points]

$$
\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & -3 & 2 & 0 & 1 & 0 \\ 3 & 1 & -1 & 0 & 0 & 1 \end{array}\right]
\xrightarrow[-3R_1+R_3]{-2R_1+R_2}
$$

$$
\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -7 & -4 & -2 & 1 & 0 \\ 0 & -5 & -10 & -3 & 0 & 1 \end{array}\right]
\xrightarrow[5R_2+R_3]{-\frac{1}{7}R_2}
$$

$$
\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4/7 & 2/7 & -1/7 & 0 \\ 0 & 0 & -50/7 & -11/7 & -5/7 & 1 \end{array}\right]
\xrightarrow{-\frac{7}{50}R_3}
$$

$$
\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4/7 & 2/7 & -1/7 & 0 \\ 0 & 0 & 1 & 11/50 & 1/10 & -7/50 \end{array}\right]
\xrightarrow[-3R_3+R_1]{-\frac{4}{7}R_3+R_2}
$$

$$
\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 17/50 & -3/10 & 21/50 \\ 0 & 1 & 0 & 4/25 & -1/5 & 2/25 \\ 0 & 0 & 1 & 11/50 & 1/10 & -7/50 \end{array}\right]
\xrightarrow{-2R_2+R_1}
$$

$$
\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/50 & 1/10 & 13/50 \\ 0 & 1 & 0 & 4/25 & -1/5 & 2/25 \\ 0 & 0 & 1 & 11/50 & 1/10 & -7/50 \end{array}\right]
\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{50} & \frac{1}{10} & \frac{13}{50} \\ \frac{4}{25} & -\frac{1}{5} & \frac{2}{25} \\ \frac{11}{50} & \frac{1}{10} & \frac{-7}{50} \end{bmatrix}
$$

b) Use A^{-1} to solve the given system. [5 points]

$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \cdot A^{-1} b = \begin{bmatrix} \frac{1}{50} & \frac{1}{10} & \frac{13}{50} \\ \frac{4}{25} & -\frac{1}{5} & \frac{2}{25} \\ \frac{11}{50} & \frac{1}{10} & \frac{-7}{50} \end{bmatrix} \begin{bmatrix} 6 \\ 14 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}
$$

(2) Determine whether or not the set of all vectors of the form $\begin{bmatrix} x \\ y \\ y-4x \end{bmatrix}$ in R^3 is a subspace of R^3. [12 points]

Let S be the set $\left\{ \begin{bmatrix} x \\ y \\ y-4x \end{bmatrix} : x, y \in \mathbb{R} \right\}$.

Then $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in S$ (with $x = y = 0$); 3

if $x, y, x', y' \in \mathbb{R}$ then $\begin{bmatrix} x \\ y \\ y-4x \end{bmatrix} + \begin{bmatrix} x' \\ y' \\ y'-4x' \end{bmatrix}$

$= \begin{bmatrix} x+x' \\ y+y' \\ (y+y') - 4(x+x') \end{bmatrix} \in S$; 3

and if $c \in \mathbb{R}$ then $c \begin{bmatrix} x \\ y \\ y-4x \end{bmatrix} = \begin{bmatrix} cx \\ cy \\ (cy) - 4(cx) \end{bmatrix} \in S$. 3

So S is a subspace of $\mathbb{R}^3$. 3

(3) Let $V = R^3$ with the addition and scalar multiplication defined by

$$(a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$
$$\alpha \cdot (a_1, a_2, a_3) = (\alpha a_1, \alpha a_2, 0)$$

Show that $(V, +, \cdot)$ is not a vector space. [12 points]

(iv) $1 \cdot u = 1 \cdot (a, b, c) = (a, b, 0) \neq u$ if $c \neq 0$.
Then $(V, +, \cdot)$ is not a vector space.

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(4) Consider the following linear system

$$\begin{aligned} 3x - y + z &= 1 \\ 2x + y - 3z &= 3 \\ x - 2y + z &= 7 \end{aligned}$$

Find the value of z by using Cramer's rule. [10 points]

Soln : We first calculate the necessary determinants.

$$\det A = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & -2 & 1 \end{vmatrix} = 3\begin{vmatrix} 1 & -3 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= (3)(-5) + (5) + (-5)$$

$$= -15$$

4

$$\det A_3 = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & 3 \\ 1 & -2 & 7 \end{vmatrix} = 3\begin{vmatrix} 1 & 3 \\ -2 & 7 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 1 & 7 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 39 + 11 - 5 = 45$$

4

$$z = -\frac{45}{15} = -3$$

2

(5) Let A be a square matrix such that $A = A^{-1}$. Show that $\det(A) = \pm 1$ [10 points]

Sol: $A = A^{-1} \Rightarrow A^2 = I$ 3

3 $\det(A) \cdot \det(A) = 1$

2 $(\det(A))^2 = 1 \Rightarrow \det(A) = \pm 1$

2

(6)

Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ -1 & 0 & -2 & 3 & 6 \\ \pi & e & \sqrt{5} & \sqrt{7} & 10 \\ 9 & \frac{\pi}{2} & \frac{e}{3} & \frac{1}{3} & 8 \end{bmatrix}$, $B = \begin{bmatrix} e & 2 & 1 \\ \frac{1}{3} & 4 & -1 \\ \frac{1}{2} & 99 & 2 \\ 1 & 7 & 1 \\ 2 & 0 & 2 \end{bmatrix}$.

Find the element $ent_{13}(AB)$. [10 points]

Answer:

$$ent_{13}(AB) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

5

$$= (1)(1) + (-1)(2) + (3)(2) + (4)(1) + (5)(2)$$

$$= 1 - 2 + 6 + 4 + 10 = 19.$$

5

(7) Find two 3×3 matrices A and B such that $AB = 0$, $A \neq 0$ and $B \neq 0$. [10 points]

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \neq 0 \qquad 4$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \neq 0 \qquad 4$$

$$AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad 2$$

(8) If A, B, C are $n \times n$ matrices and α is a scalar, then which of the following is **always True**? [7 points]

(a) $\alpha A(BC) = A(\alpha B)C = AB(\alpha C)$

(b) $A^2 - B^2 = (A - B)(A + B)$

(c) $(A + B)^2 = A^2 + 2AB + B^2$

(d) $AB = AC \Rightarrow B = C$

(e) $(AB)C = A(CB)$

Correct : (a)

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(9) Which one of the following statements is **True**? [7 points]

(a) every diagonal matrix is invertible.

(b) every upper triangular matrix is invertible.

(c) every lower triangular matrix is invertible.

(d) every nilpotent matrix (i.e. $A^n = 0$ for some $n \geq 1$) is invertible.

(e) every matrix A with $A^2 = I$ is invertible.

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Correct answer is (e), $A^2 = I$, Then $A^{-1} = A$.

→ Diagonal matrix is not invertible if one element entry of the main diagonal is zero.

→ Upper triangular matrix (and lower triangular matrix) is not invertible if one entry in the main diagonal is zero.

→ A Nilpotent matrix is never invertible since If A is invertible and $A^n = 0$

$A^{-1} A^n = A^{-1} 0 \Rightarrow A^{n-1} = 0$. Again

$A^{-1} A^{n-1} = A^{-1} 0 = 0 \Rightarrow A^{n-2} = 0$

Thus $A^2 = 0$ and $A^{-1} A^2 = 0 \Rightarrow A = 0$

absurd (since the zero matrix is not invertible).

(10) If A and B are 4×4 matrices such that $\det A = 4$ and $\det B = 5$, then the value of $\det(AB) - 16\det(A^{-1})$ is [7 points]

(a) 16

(b) 19

(c) 5

(d) 21

(e) $\frac{39}{2}$

$$\det(AB) - 16\det(A^{-1}) =$$

$$= \det(A)\cdot\det(B) - \frac{16}{\det(A)}$$

$$= (4)\cdot(5) - \frac{16}{4}$$

Correct

$$= 20 - 4 = 16 \Rightarrow (a)$$

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